Nonaxisymmetric Magnetic Structures in RS Canum Venaticorum–type Stars

Pro Gradu Thesis

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Contents

Chapter 1

Introduction

In the 20th century a vast amount of evidence for solar–like outer atmospheres in other cool stars than the Sun was collected by studying emission lines caused by some nonradiative processes. In the Sun this kind of emission seems to be closely related to magnetic phenomena. It is because of this solar relation that various emission lines originating from stellar chromospheres, transition regions and coronae are widely used as proxy indicators of magnetic phenomena in late–type stars.

One kind of widely used proxy for stellar magnetic fields has been stellar photometric variations. In 1930's it was discovered that there is a migrating distortion wave on the light curves of some spectroscopic binaries. The straightest explanation was starspots. This view has been slowly becoming more generally accepted. The wave is now observed in many kinds of cool stars. With the modern surface imaging methods the stellar surface inhomogeneities causing the variation can be mapped using the observed asymmetries in light curves and spectral line profiles.

Broadening of spectral lines interpreted to be caused by Zeeman effect gives somewhat more direct evidence of solar–like magnetism on other cool stars. Earlier measurements of magnetic field strength and filling factor were probably related to plage– and active network–like bright features. There is now some spectroscopic evidence that part of this broadening originates from dark sunspot–like regions. In the spectra of some stars there are clear signs of fragile molecules like TiO which cannot survive at photospheric temperatures in these stars. One solution to this dilemma is that this molecular absorption originates from sunspot–like magnetically cooled regions.

The solar rotation and quasiperiodic magnetic cycle has long been a challenge to the astrophysics. It was thought that the solar equatorial acceleration could be a fossil remnant from the earlier times. The fossil interpretation of the periodic solar magnetism is harder if not impossible to accept. Modern observations show that there exist also in other stars phenomena whose timescales and other characteristics are comparable to solar magnetic processes. One of the most important clues are any stellar observations pointing to periodic variability analogous to solar magnetic cycle.

As the thermal instability and turbulent nature of the outer layers of the Sun and other cool stars became apparent the fossil hypotheses about solar equatorial acceleration and magnetic cycle became almost impossible to believe in. This is because turbulent eddies enchance the decay rates of any large scale structures present in the atmospheres of cool stars by many orders of magnitude. Yet the solar differential rotation and magnetic cycle has been observed to exist in the Sun for some centuries basically unchanged. The key to this dilemma is the remarkable finding that the small-scale turbulent motions need not always be a cause of destruction, but they can also give rise to some large-scale structures.

It is through the anisotopic turbulence induced by solar rotation and by gravitation (Λ-effect) that the solar equatorial acceleration or more generally differential rotation comes about. Coriolis forces induced by solar rotation are essential in α-effect. With the Λ-effect and α-effect the solar cycle can be explained quite easily. Any poloidal magnetic fields in the Sun are seed fields for the toroidal magnetic fields generated by the latitudinal differential rotation. The α-effect closes the cycle by regenerating the poloidal field from the toroidal field.

Long–term photometric time series and surface maps of active late–type stars, such as the RS Canum Venaticorum binaries and FK Comae Berenices–type stars, show strong nonaxisymmetric distribution of active regions, unlike the dominantly axisymmetric solar field. There are usually two active longitudes separated by 180°. Many properties of these longitudes over timespan of decades can be studied with the help of migration curves. One of longitudes is usually more active than the other at a given moment. The change of the activity level between the longitudes has been shown to be cyclic in some stars with a period of the order of ten years.

In this work nonaxisymmetric dynamo models are developed for active late–type stars, solving both for the mean–field dynamo and Reynolds equation including several non–linearities. The main aim was to find out if dominating nonaxisymmetric oscillatory modes of the magnetic field can be exited with rapid rotation. In this thesis both the literature describing the properties of the RS Canum Venaticorum binaries and some other chromospherically active star groups is reviewed. Also the basic magnetohydrodynamic equations from which the dynamo and Reynolds equations can be derived are reviewed.

Chapter 2

RS Canum Venaticorum phenomena

2.1 The subgroup of Algols with Ca II H and K emission

According to Weiler (1978) a possible sub–group of the traditional Algol binaries was first noted by Struve in 1946. The systems exhibited strong H and K emission lines of calcium with the widths of the emission comparable to those of the absorption lines. According to Hall (1972) Plavec and Grygar pointed out in 1965 that among a sample of 37 Algol–type eclipsing binaries, five (RS CVn, WW Dra, Z Her, AR Lac, and SZ Psc) appeared to form a distinct group with similar behaviour. Three of these were among the 14 eclipsing systems with “undersize“ subgiant secondaries listed by Kopal in 1959. The most complete list of these binaries at the time were published by Popper in 1970, who considered H and K emission as the principal identifying characteristic. In 1971 Oliver summarized the physical, spectroscopic, and photometric peculiarities which characterized this group. Hall (1972) remarked that: “RS Canum Venaticorum is of interest because it suffers from a greater variety of complications than almost any other eclipsing binary known to date. RS CVn is representative of, and perhaps should be considered a prototype of, the puzzling group of detached double–subgiant systems . . .”

Hall (1976) proposed a working definition of ‘RS CVn system’ if a binary fulfils the following characteristics:

- The spectral type of the hotter component is F or G and a luminosity class IV or V.
- Strong Ca II H and K reversal seen in the spectrum.
- The orbital period is between one day and two weeks.

With these characteristics the 1976 list of RS CVn binaries included 24 members. In it some systems with orbital period larger than 14 d were included such as λ And (G8 III–IV + ?; $P = 20.5$ d) and α Aur (G0 III + G5 III; $P = 104.0$ d). For this reason it later seemed be more convenient to adopt a definition based on the presence of activity along with other properties, instead of the orbital period.

Hall also gave two more characteristics seen in large portion – but not in all – of the systems classified as RS CVn:

- The H and K emission arises either from the cool star or from both components.
- The presence of a distortion in the light curve outside of eclipse.

The second property, considered as fundamental to define a RS CVn system, suggests the presence of dark spots of magnetic origin on the surface of the active star by solar analogy (Montesinos et al. 1988).

2.2 The discovery of some related spectroscopic binaries

2.2.1 UX Arietis

The noneclipsing binary HD 21242 (UX Ari) was discovered to have strong H and K emission by Hogg in 1939. It was found to be a spectroscopic binary by Popper in 1954 (Carlos and Popper 1971). Carlos and Popper (1971) published a spectroscopic orbit (period 6.4 d) for UX Ari. Absorption lines in the photographic region and emission lines of Ca II showed opposite velocity shifts. Close inspection revealed some absorption lines of a star of later type, and weak Ca II emission lines in phase with the strong absorption. The stars were tentatively classed as G5 V and K0 IV. The subgiant category was assigned to the second component by analogy with eclipsing binaries with similar spectra. The component, which has the strong Ca II emission, dominates the spectrum in the visual region. It was shown to be the more massive by a small but definite ratio.

Spectroscopically, UX Ari was indistinguishable from a number of the 25 eclipsing systems known with H and K emission in one or both components. Carlos and Popper (1971) noted that there must be many similar noneclipsing systems. They considered that HD 118216 (HR 5110, BH CVn) studied by Conti in 1967, might also belong to the group. Hall (1972) mentioned that UX Ari should be probably added to the RS CVn group.

After the detection by Hjellming and Blankenship in 1973 of radio emission from AR Lac, itself an RS CVn–type binary, it was suggested that they look at UX Ari (Hall et al. 1975). This suggestion resulted in the discovery of radio emission by Gibson, Hjellming, and Owen in 1975. According to them UX Ari was the second most active radio star (after Algol) in terms of flux density and degree of erratic activity.

This discovery prompted Hall et al. (1975) to make photometric observations on UX Ari. The light seemed to vary with the orbital period determined spectroscopically by Carlos and Popper (1971). The light curve was roughly sinusoidal, although somewhat skewed. Eclipses were almost certainly not producing this light variation because the light curve was not characteristic of an eclipsing variable and because the minimum of the curve occured at orbital phase 0.1 instead of 0.0 (Hall et al. 1975).

2.2.2 V711 Tauri

Bopp and Fekel (1976) showed HR 1099 (ADS 2644A, V711 Tau), a G–type system with strong Ca II H and K and Hα emission, to be an RS CVn binary system with orbital period of 2.84 d. The system beared a striking resemblance to previously discovered noneclipsing RS CVn system UX Ari.

Photometry by Landis and Hall from 1976 showed no eclipses, but a quasisinusoidal variation $\Delta V \sim 0.1$ mag with period 2.822 d was evident. Landis and Hall remarked that the light curve of V711 Tau was virtually identical to that of UX Ari in 1972. The Ca II H and K intensity of the primary component matched closely the strong H and K emission in V711 Tau. Hα spectra showed V711 Tau to be very similar to the RS CVn system UX Ari. Both stars showed Hα emission, with about the same intensity.

The above similarities, along with the radio detection of UX Ari suggested that V711 Tau was a likely candidate for radio observations. Owen reported in 1976 that V711 Tau was detected at 2695 and 8085 mHz with flux densities ranging from 10 to 170 mJy. Flare–like behavior similar to that seen in UX Ari was noted. Owen noted that the level of radio activity in V711 Tau might be significantly higher than that in UX Ari. In the latter star, the strongest radio flare seen peaked at ~ 160 mJy. The observations extended over several months. A more intense flare was seen from V711 Tau after only a few days of observations.

Bopp and Fekel (1976), however, noted some differences between these systems. In UX Ari, the equivalent width of the Hα emission was quite variable but no profile changes were found. Observations showed no convincing evidence for variation in Hα equivalent width, but the profile was observed to change radically by the very broad Hα emission seen near one particular orbital phase. Bopp and Fekel (1976) remarked that this behavior can be phase dependent, or it may be a result of sporadic surface activity on the primary component. Including Wilson's data from 1964, two out of eight Hα spectrograms showed this peculiarity. It was concluded that the circumstances producing this unusual profile must occur reasonably often.

2.2.3 Rucinski's spot–covered red star II Pegasi

According to Rucinski (1977) the variability of HD 224085 (BD +27°4642, Lalande 46867, II Peg) was discovered by Eggen in 1968 on the basis of five discordant observations in V. These photometric changes and the fact that the star had been known as a single–line spectroscopic binary with a period of 6.72 d were apparently the only basis for Taylor to classify the system as an eclipsing binary in 1970. Chugainov obtained the first light curve in 1976 and determined the photometric period to be 6.75 d. The changes of light were observed to be continuous and no eclipse was noted. Chugainov interpreted the variations as arising from large stellar spots as in BY Dra–type stars. He found much support for this idea in the asymmetry of the light curve and in the well–determined change of its amplitude from about 0.1 mag to more than 0.3 mag between the two observing seasons. Chugainov overlooked the known binarity of the star.

Rucinski (1977) noted that II Peg had relatively early spectral type for a spot–covered red dwarf. The mean of earlier classifications was about K1 which was earlier than any other known BY Dra–type star. The star had been also classified as a K2 giant in 1968 by Eggen. Rucinski (1977) noted also that during two years since Chugainov's photometrical observations in 1974, the light curve had undergone three major changes: in its asymmetry, in the average level dropping by about 0.10 – 0.15 mag, and in the shift to the left relative to the orbital phases. The large asymmetry of the light curve and its changes suggested a highly nonuniform longitudinal distribution of spots over the stellar surface. This distribution seemed to be more or less constant in the time scale of weeks or months, but the spots had to be able to appear, rearrange themselves, or decay over periods of a few months or years. The general drop in brightness was probably related to these changes. The shift to the left could indicate that the photometric period, probably related to the stellar rotation, was slightly different from the orbital one, as in the RS CVn–type stars. Rucinski (1977) determined a relatively fast migration rate of 1100 or 300 orbital periods for the photometric wave. This was within the range of migration rates (350–6000 orbital periods) observed for the RS CVn–type stars.

Rucinski (1977) noted that II Peg seemed to share most of the properties of the BY Dra–type stars: the timescales of spot formation and rearrangement, and the tendency for some flaring activity in the ultraviolet were found to be similar. The spectral type, however, was earlier than any of the BY Dra–type stars, and the star seemed to be slightly above the main–sequence. The infrared excess observed by Taylor in 1970, the migration of the photometric wave, and the spectral type close to K0 suggested that the visible component of II Peg is similar to the cooler components of the RS CVn–type stars. (Rucinski 1977).

Rucinski (1977) observed the definitely present weak line λ 6707 Å of lithium. This gave at the time support for the possibility of II Peg being a "post–T Tau star".

However, Fekel and Balachandran (1993) and Randich et al. (1994) found the presence of substantial amounts of Li to be typical in many chromospherically active stars. Now the question is whether these relatively high Li abundances are due to enhanced chromospheric activity or rather are a consequence of the evolutionary history of the stars.

Bopp and Noah (1980) did not find the behavior of II Peg to be necessarily a indication of BY Dra or pre–main–sequence variability. Spectroscopy showed the Hα emission feature to be variable in equivalent width by a factor of ten on time scales of several days. Most of this variability was in phase with the stellar rotation period, suggesting strong localization of Hα emitting regions. Sudden flare–like enhancements of Hα were seen which exhibited decay times of days. Noticeable red asymmetry in the Hα profile was reminiscent of the emission profile exhibited by the RS CVn binary V711 Tau. The observed Hα profile and equivalent width variations seen in II Peg were very reminiscent of the behavior of this feature in the RS CVn binaries UX Ari and V711 Tau. The slow variation of Hα, modulated by stellar rotation period, was previously seen in UX Ari, and also in dK7e BY Dra variable CC Eri. This suggested a strong localization of Hα emitting regions on the stellar surface.

The sudden enhancements of Hα in II Peg, which showed no apparent phase dependence, were also seen in some RS CVn binaries. Radio flares on V711 Tau and HK Lac, for instance, were observed to be accompanied by enhanced Hα emission which decayed slowly over an interval of days. Similar Hα behavior was reported for the longer period system HK Lac. Such behavior were not seen in the BY Dra variables. Though optical flares (often accompanied by radio emission) are seen, any enhancement of the Balmer emission lines decays on the time scales of minutes or hours. The Hα emission profile in II Peg closely approximated the Hα profile of V711 Tau. The Hα profile of V711 Tau was known to be variable. Several authors described the feature as broad (FWHM $\approx$ 3 Å – 4 Å) and often asymmetric towards the red. Normally there was no suggestion of a central reversal. In contrast, high–resolution spectra of dMe stars (many of which are also known BY Dra variables) all show Hα emission profiles that are relatively narrow (FWHM $\approx$ 1 Å), symmetric, and have central reversals. Thus Bopp and Noah (1980) concluded that from the spectroscopic point of view, II Peg do not closely resemble a BY Dra variable.

Clearly II Peg did not spectroscopically resemble the post–T Tau object FK Serpentis either. The Hα in FK Ser is several times stronger than that seen in II Peg or any other RS CVn star (EW $\sim$ 6 Å – 10 Å). In addition, Hα in FK Ser showed a blueshifted absorbtion component. The Li I line had EW $\sim$ 600 mÅ, an order of magnitude stronger than the feature in II Peg.

Bopp and Noah (1980) concluded that the spectroscopic characteristics of II Peg most closely match with an RS CVn binary, such as V711 Tau, rather than with a BY Dra or post–T Tau star.

2.3 Evolutionary status of the RS Canum Venaticorum binaries

Hall (1972) commented that the evolutionary status of the RS CVn binaries certainly was not clear. Plavec concluded in 1967 that this group represents a category of binaries, quite possibly of different origin and evolution than the semidetached binaries of Algol type. In 1971 Oliver emphasized that many of the characteristics of this group were not easily explainable on the basis of existing data and theory. In order to explain a typical RS CVn system, with both components lying above the main–sequence as subgiants, Hall (1972) considered three rival interpretations: post–main–sequence, both before and after mass exchange, and pre–main–sequence. Catalano and Rodono had considered in 1967 the possibility that the cool star in RS CVn is in pre–main–sequence contraction. In 1969 Ziolkowski suggested parenthetically that this may be the case for all of the RS CVn binaries. Based on the evidence then available the pre–main–sequence interpretation seemed to be the most attractive one.

Hall (1972) noted that the general feature of the RS CVn binaries seems to be that they are detached, some by large margin. Detached binaries are difficult to explain as remnants of post–main–sequence mass exhange in a close system. Paczynski in 1967 and Refsdal and Weigert in 1969 suggested that the subgiants in such systems had already ignited helium and had subsequently become detached for that reason, but Hall (1972) considered the theoretical models to be all too hot. The individual components of most RS CVn binaries have masses around 1 $M_\odot$ and hence should take around 10×10^9 years to begin evolving off the main–sequence. Hall (1972) noted that the bulk of the stars in our galaxy, those in the disk population, are only of $1 - 8 \times 10^9$ years old.

The strong H and K emission in RS CVn stars seemed to be difficult to explain by any post–main–sequence interpretation. Hall (1972) noted that strong H and K emission is one of the most outstanding spectroscopic characteristic of the T Tau stars. He admitted that the emission line spectrum of the cool star in RS CVn did not resemble that of the T Tau stars closely enough that it could be classified as such by the spectroscopic criteria of Herbig from 1962. In line with the correlation between chromospheric activity and age suggested by Wilson (1963), RS CVn could be considered to be a very young star, intermediate in age between a T Tau star and a late–type dwarf emission star (Hall 1972). Also the mass loss rate of 10^{-6} $M_\odot yr^{-1}$ estimated for RS CVn seemed to be in line with the $0.3 - 5.8 \times 10^{-7}$ $M_\odot yr^{-1}$ range of mass loss observed by Kuhi in 1964 in six bright T Tau stars.

Hall (1972) considered all the photometric complications in the light curve of RS CVn well accounted for by his model of solar–like magnetic cycle. There was thus no evidence of the large irregular fluctuations in brightness characteristic of the T Tau stars. Poveda had suggested in 1965, that the large irregular fluctuations in brightness observed in T Tau stars are not intrisinc variations in the star itself but are due to a complex sequence of eclipses by dense clouds of planetesimals orbiting around the T Tau star in a circumstellar disk.

Few special cases indicated that the pre–main–sequence scenario for RS CVn systems could be wrong. Hall (1975) noted that ADS 10152 AB is a visual binary of which the brighter component is the eclipsing RS CVn–type binary WW Dra. According to Popper the minimum masses for the two components of WW Dra are 1.35 $M_{\odot}$ and 1.35 $M_{\odot}$. Hall (1975) estimated the mass of ADS 10152 B to be 1.1 $M_{\odot}$. Hall (1975) noted that if WW Dra is in pre–main–sequence evolution, the less massive F8 V visual companion should be still in pre–main–sequence contraction. It also should be redder and further above the main–sequence than WW Dra. He observed neither of these expectations: ADS 10152 B is bluer and it is on the main–sequence.

Bopp and Fekel (1976) noted that the precense of the visual companion, ADS 2644B, puts some interesting constraints on the evolution of RS CVn system V711 Tau. Various determinations of relationships between H and K intensity and stellar age, implied to Bopp and Fekel (1976) an age of order 2×10^9 yr for ADS 2644B and to its orbiting system V711 Tau. This derived age clearly showed that the pre–main–sequence stage of evolution argued by Hall (1972) was not valid in this case.

Popper and Ulrich (1977) gave some further arguments against the pre–main–sequence scenario:

- The RS CVn stars are not physically associated with regions of known star formation activity.
- The lifetime of post–main–sequence stars is 100 times as great as the pre–main–sequence lifetime among a sample of stars of a specified mass and radius range.
- In most of the systems the more massive star has the larger radius, as expected for post–main–sequence systems, but not for the pre–main–sequence systems.
- There are main–sequence companions of lower luminosity than the primaries. These stars could not have reached the main–sequence before the more massive stars.

Because of the long interval required for evolution of some low–mass systems, loss of mass from some systems seemed also to be required.

Popper and Ulrich (1977) noted that the RS CVn systems do not differ markedly from the nonemission detached group in their orbital angular momentum per unit mass, while the semidetached and contact systems, as expected, have less angular momentum. All the RS CVn systems with radii determined were detached. They had mass ratios within 30% of unity, as did all the detached nonemission systems.

The most striking result was the nearly complete separation of the two groups of detached systems when plotted on the mass–color, mass–radius, and color–luminosity planes. The nonemission, well–behaved systems occupy principally the main–sequence band, while the RS CVn systems occupy primarily the Hertzsprung gap. The less evolved components of RS CVn stars are evolving off the main–sequence band. The more evolved are situated on the knee before the rise to the giant branch.

Popper and Ulrich (1977) considered this to provide strong circumstantial evidence that the components of RS CVn systems have evolved through the main–sequence band. A system would develop RS CVn characteristics when the stars have completed hydrogen burning in their cores and have developed convective envelopes. For most of the systems, the course of the evolution does not appear to differ radically from that expected for single stars of the same masses. In most of these RS CVn systems the more massive star is the more evolved.

Morgan and Eggleton (1979) presented calculations which supported the point of view of Popper and Ulrich (1977). They concluded that not only the evolutionary status of the components of RS CVn binaries is due to normal nuclear evolution, but that these binaries are the most normal evolved eclipsing binaries around. They have just the characteristics that detached binaries need to have in order to maximize the probability of their being found to eclipse. That conventional Algols outnumber them considerably in the observed sample of eclipsing binaries is due to the peculiar evolutionary history (considerable mass transfer) of conventional Algols, and not to any peculiarity in the evolutionary history of RS CVn binaries.

Four important properties which Morgan and Eggleton (1979) considered to have the most direct bearing on the evolutionary status of RS CVn stars, are:

- Mass ratios are usually very close to unity, the cooler, larger component being typically 5 % more massive; normal Algols have mass ratios which are typically very far from unity.

- The cooler star usually has a spectral type fairly close to K0 III–IV: the cooler components in normal Algols are usually somewhat earlier, and show greater range.

- The hotter star has a spectral type in the range F4–G5 IV–V: normal Algols usually contain earlier stars, B–A5 V.

- The cooler star in most RS CVn systems is clearly smaller than its Roche lobe (by about 10–50 %), and does not appear to be ejecting mass into a ring about its companion: in normal Algols such mass transfer is generally thought to be taking place.

Morgan and Eggleton (1979) noted that the probability that some particular kind of binary star will be well represented in a sample of eclipsers depends on three main factors.

(1) One star should be both larger and less luminous than the other so that if eclipses occur, the primary one will be reasonably deep.

(2) The larger star should subtend the largest possible solid angle at the other, i.e. it should be close to filling its Roche lobe, or actually filling it.

(3) Both stars should be in fairly long–lived evolutionary states. Otherwise the particular kind of binary will be intrinsically rare.

At least two of these conditions are satisfied, very fortuitously, by the kind of star, like Algol itself, which has undergone and is still undergoing mass transfer, and now has a mass ratio of about 0.3 with the initial primary less massive. For this case Morgan and Eggleton (1979) listed the following circumstances.

(a) The initial secondary, which would have been smaller and less luminous than its companion when both stars were on the main–sequence (so eclipses could not have been deeper than 0.75 mag, and would usually have been much shallower), is now more massive and much more luminous, often more luminous than the evolved original primary, yet is substantially smaller; so eclipses can be deep, sometimes over 4 mag.

(b) The original primary, having necessarily filled its Roche lobe in the past, is unlikely to be far from it now (because stars generally grow larger as they evolve, and contract quickly to a very small radius once they detach from the Roche lobe). Thus it will subtend about the largest possible solid angle at its companion.

(c) Theoretically the timescale of mass transfer can ultimately settle down to a nuclear time–scale. Thus the binary may spend an appreciable fraction of its nuclear lifetime in the Algol–like phase. It remains to confirm this observationally.

These favourable circumstances are due to mass transfer. Morgan and Eggleton (1979) discussed also the situation without mass transfer, i.e. systems which have not yet evolved to mass transfer. They started discussion with two zero age main–sequence stars that are well detached. Prior to Roche lobe overflow the more massive star will be the more luminous, and will become even more luminous as it evolves. Thus one might suppose that eclipses would never be very prominent, $\lesssim 0.75$ mag. However there is a possibility of deeper eclipses in rather special circumstances, if the primary has evolved as far as the local minimum luminosity at the base of the giant branch while the secondary has evolved to the local maximum of luminosity just after hydrogen exhaustion. In these maximal eclipse depth binaries the eclipse can be over a magnitude deep. For primary masses below ~ 2 M$_\odot$, corresponding to ages $\gtrsim 10^9$ yr, the star's evolution slows considerably at the base of the giant branch as the core becomes degenerate and is able to support the star on a nuclear time–scale. Thus only for fairly low masses, and for mass ratios fairly close to unity, both conditions (1) and (3) above are satisfied.

Morgan and Eggleton (1979) found condition (2) above to be little harder to quantify. If the separation, or equivalently the period, is too small $\lesssim 2$ d it is impossible for a binary to get to the maximal eclipse depth configuration described above before Roche lobe overflow of the more evolved star occurs. There will also be a drop in probability of finding such systems with periods $\gtrsim 10$ d, because for long periods, i.e. wide separations, a star at the base of the giant branch would be too small in relation to its Roche lobe to give much probability of eclipse. Alternatively, if in such a wide binary the more evolved star nearly fills its Roche lobe, it must be some way above the base of the giant branch. Its evolution there will be more rapid, so such systems will be rarer. At the same time the more evolved star will be brighter relative to its companion, so eclipses will be shallower. Morgan and Eggleton (1979) concluded that eclipses are most likely if the larger, cooler star nearly fills its lobe, and so the maximum eclipse depth systems should be found preferentially in a period range of about 2 – 10 d.

2.4 RS CVn phenomena and BY Dra syndrome

In the light of the substantial amount of accumulated data on many newly identified chromospherically active stars, Fekel et al. (1986) rose the question, do the defining characteristics of the classes of such stars need to be modified? They noted that the active–chromosphere phenomenon is seen in a number of classes of late–type stars, including RS CVn binaries, BY Dra variables, FK Com stars, and W UMa binaries. Although the W UMa stars were certainly a distinct class of stars, it was not obvious to which of the other classes an active star belongs, since many of the characteristics of the remaining classes are similar. Eggen had questioned already in 1978 whether these classes are too restrictive and artificial, since strong chromospheric activity appears in a wide variety of late–type stars.

Hall (1978) noted that the RS CVn properties are almost certainly related closely to the BY Dra syndrome observed in the dMe flare stars, although the flare stars do not all seem to be binaries. Popper (1980) noted that systems being referred to as "RS CVn" include a wide variety in which a number of interrelated phenomena occur in various combinations. He pointed that class of systems should probably have common evolutionary status. He even remarked that it might be wise to refer to RS CVn phenomena rather than to RS CVn systems.

Fekel et al. (1986) noted that Hall had defined in 1976 the RS CVn binaries from a rather limited data set, consisting primarily of late–type eclipsing binaries with H and K emission lines. Since nearly all were eclipsing, they had relatively short (1–14 d) periods. In addition, nearly all of these systems contained a less evolved, hotter component. These "classical" RS CVn binaries were supplemented by a long–period ($P \gtrsim 14$ d) group. At the time, the relationship of this long–period group to the classical RS CVn binaries was uncertain. According to Fekel et al. (1986) there is no substantial difference between the classical and the long–period groups. The longer period simply enables the more massive star to attain a greater radius and luminosity before the binary becomes a mass transfer system.

From some of the arguments above, Fekel et al. (1986) suggested the use of the three following properties to define a RS CVn system:

- At least one star must show intense emission in the H and K lines of Ca II.
- The system must present periodical variations in the luminosity, but not due to pulsation, eclipses or ellipticity.
- The more active star must have a spectral type F, G or K, subgiant or giant, i.e. be in a evolved evolutionary state.

So more attention is thus paid to evolutionary considerations and to the presence of activity in the components of the systems, instead of, for instance, constraining the inclusion of a given star into the group to a restricted orbital period interval. According to the third point of this new definition, at least 7 of the 69 systems given by the 1981 list of Hall seemed not to be members of the RS CVn family, because both their components had luminosity class V (Montesinos et al. 1988).

Collier noted in 1982 that most of the short–period RS CVn systems should be classified as early–type BY Dra systems. Fekel et al. (1986) proposed to expand the definition of BY Dra stars to include F and G dwarfs as well. Since these stars are main–sequence stars, it seemed appropriate to put them in the BY Dra class.

2.5 Chromospherically active binary stars

Strassmeier et al. (1993) noted that there is now a large sample of stars with activity levels exceeding that of the "active" Sun by several orders of magnitude. These "chromospherically active" stars include single and binary stars as well as pre– and post–main sequence objects and have rapid rotation and deep convective layers in common. The class of RS CVn–type binaries was defined by Hall in 1976 along with five sub–classes of "related binaries": the long–period group, the short–period group, the flare–star group, the group similar to V471 Tau, and the W UMa group. Strassmeier et al. (1988) noted that although certain aspects of Hall's definition has been criticized since then, it proved extremely useful in drawing attention to the whole array of what subsequently has become known as extreme chromospheric activity but which at the time was virtually unrecognized. Prior to 1975 such activity was blamed incorrectly on gas streams, circumstellar rings, pulsation, precession, tidal distortion, heating by the interstellar medium, and orbital motion around third companions. Star spots were mentioned only rarely and generally dismissed as irrelevant.

Because of the huge amount of available observational material, a summary of the properties of these stars seemed to be needed. Therefore, three independent unpublished RS CVn catalogues were compiled. Strassmeier et al. (1988) presented a catalogue which combined these three RS CVn data bases and extended it to include the BY Dra binaries as well as other non–classified chromospherically active binary stars. Strassmeier et al. (1988) noted that several binaries can legitimately be classified as members of both RS CVn and BY Dra types. Certain obvious chromospherically active binaries cannot be legitimately classified as either. That is why the catalogue of chromospherically active binary stars focused on chromospheric activity itself as the prime criterion for inclusion.

The two prime criteria for including a star in this catalogue were: the object must be a binary and at least one component must be chromospherically active. Chromospheric activity was defined by the presence of Ca II H and K emission lines (Strassmeier et al. 1988). Other restrictions to sample were made when the chromosphere of the active component(s) was obviously influenced by other physical processes, like pulsation or common convective envelope. Therefore binaries containing M–giants and all W UMa type binaries were excluded. Another class of chromospherically (very) active stars, the FK Com stars, were not included in the catalogue because they are per definition single stars and no velocity variations have been found. The same applied for the pre–main–sequence T Tau stars. Although chromospherically active, they probably all are single stars (Strassmeier et al. 1988).

The catalogue contains 168 chromospherically active binary stars. It contains information on the photometric, spectroscopic, orbital and physical properties of the systems as well as the space motions and positions. In a canditate list there are additional 37 stars having similar characteristics, but they were not definitely known to be binaries or no H and K emission was observed.

The amount of information on the individual systems and the number of newly discovered active binaries became so large that an expanded and updated edition of the catalogue was prepared by Strassmeier et al. (1993). It contained 206 active binaries and 138 candidates. The two prime criteria for including a star in this catalogue were not changed from the first edition. The object had to be "close" binary and at least one component had to be chromospherically active. Presence of Ca II H and K emission lines and/or abnormally high absolute emission–line surface flux was considered to be indication of "chromospheric activity".

This extension from solely visual based inspection of spectra to include also absolute Ca II H and K emission–line surface fluxes was motivated by several reasons.

- Rapid rotation will broaden an emission line and thereby weaken its strength.
- In the presence of a hot companion the emission from a cool, active component will be diluted and thus underestimated or even judged inactive.
- Absolute flux places all stars on a uniform scale relatively independent of spectral resolution.

It is also easier to set a numerical limit for the decision whether to include a particular star or not. For the definition of the "cut–off" Strassmeier et al. (1993) used an activity parameter R_{CaII} which is the radiative loss in the Ca II H and K emission lines in units of the bolometric flux. Strassmeier et al. (1993) included a star in the catalogue when $R_{\mathrm{CaII}} > 2 - 4 \times 10^{-5}$. This limit followed from work of Strassmeier et al. (1990). They identified the well–known Vaughan–Preston gap running at $R_{\mathrm{CaII}} \sim 2 - 4 \times 10^{-5}$ in their study of 100 active and inactive single and binary stars of spectral type F6 to M2 and luminosity class III, IV, and V. This Vaughan–Preston gap was discovered to divide the northern field stars to the groups of active and inactive stars (Vaughan and Preston 1980). The southern solar type field stars are divided similarly (Henry et al. 1996).

Chapter 3

Stellar nonaxisymmetric magnetic structures

3.1 The wave

The hypothesis that non–uniform distribution of regions of constant brightness, coupled with stellar rotation, could be responsible for stellar photometric variations, was proposed as early as the seventeenth century (Kopal 1982). In 1667 Montanari had discovered the large change in brightness of Algol (β Persei) which was to become the first known photometric binary. J. Goodricke considered in 1783 two scenarios for this variability. Either there were big spots on the stellar surface, or the star was eclipsed by a gigantic 'planet'. Since neither sunspots nor planets even of approximately matching size were known from the solar system, neither of his ideas found much approval. The hypothesis of a 'stellar eclipse' gradually gained recognition when it became evident that this was a capable explanation of the precise recurrence of the light variation (Heintz 1978). It is very interesting to note here that later some of the most convincing evidence for starspots arose from the RS CVn binaries. These were first identified as a distinct subgroup of traditional eclipsing Algols as discussed in Section 2.1. At this time, when the effects related to stellar eclipses had been clarified to some extent, the residual variation could indicate nonuniformities in the stellar surfaces. So nowadays it seems that actually both Goodrickes hypothesis (stellar eclipses and starspots) are needed simultaneously to explain the variability of these stars.

Some mathematical aspects of 'spot theories' were developed by Bruns in 1882. In 1906 Henry Norris Russel confirmed (independently of Bruns) that, for an arbitrary distribution of brightness over the surface of a star, the underlying photometric problem is essentially indeterminate – in the sense that the axial rotation of a star with arbitrary distribution of brightness on the surface can account for any observed light curve of stellar variability. (Kopal 1982). It has been mainly because of this indeterminacy why the study of starspots has not been generally considered to be very fruitful. Another point is the fact that only very recently has some direct spectroscopic evidence been gathered for sunspot–like inhomogeneities on the stellar surfaces. Some of this evidence is reviewed in Section 3.4. As early as in the 1930s light variations on timescales of months and which were not attributable to tidal distortion, reflection, or eclipse effects were detected in the late–type eclipsing binaries RS CVn (Sitterly in 1930) and AR Lac (Himpel in 1936; Wood in 1946; Kron in 1947) (Fekel 1983). Sitterly assumed that the distortion in the light curve of the RS CVn binary could be due to photospheric spots (Godoli 1976). Kron interpreted these variations as a nonuniform surface brightness distribution which rotated in and out of view. Since then it has been found that this phenomenon occurs in the vast majority of late–type spectroscopic binaries which have short or moderate periods. These systems are generally divided into two groups, the BY Dra variables and the RS CVn variables (Fekel 1983). More generally this kind of photometric variability seems to be correlated with the mean chromospheric activity level of the star in question (Radick et al. 1990). Since the mean chromospheric activity level correlates with the stellar rotation rate it is feasible possibility that the amplitude of the wave also depends on the rotation rate. Hall (1991) argues that the Rossby number a little less than unity is a sharp dividing line between the non-variable (wave amplitude less than 0.01 mag) and the variable stars (wave amplitude more than 0.01 mag).

3.2 The 23.5 year spot cycle on RS Canum Venaticorum

Hall (1972) developed a simple model to reproduce some of the complex behaviour observed in the light curve of eclipsing binary RS CVn. A huge region of sunspot–like activity is darkening the surface of the cooler star. The region extends only about half way around the star, covering a range of about 180° in longitude. The activity is periodic, with 1800 orbital cycles between successive maxima. Thus there is a "sunspot cycle" of about 23.5 years in the cool star of RS CVn.

Hall (1972) listed the following complications in the light curve of eclipsing binary RS CVn.

- A wave–like distortion which perturbs the light curve outside eclipse, causing it to reach one maximum and minimum each orbital cycle.
- Secondary minimum is about half as deep as one would expect from the relative surface brightnesses of the two stars as deduced from their color indices.
- The wave, while maintaining its shape, migrates year by year towards decreasing orbital phase.
- The depth of primary minimum varies with time.

- Secondary minimum does not always fall midway between adjacent primary minima, sometimes occuring early and sometimes late.
- Migration rate is not uniform.
- The amplitude of the wave is not constant, having been as large as $\Delta V = 0.2$ mag from the maximum to the minimum of the wave and at times so small as to render the wave almost unnoticeable.
- The orbital period undergoes large and seemingly erratic changes, both increases and decreases, despite the fact that both stars are well within their respective Roche lobes (thus excluding any Algol–like mass transfer).

According to Hall (1972) Gaposchkin in 1939 was the first to conclude correctly that the cooler star in system of RS CVn must be causing the wave. Hall (1972) considered the wave, which reaches one maximum and one minimum each orbital cycle, to be a simple consequence of the spotted cooler star rotating once each orbital cycle.

From the approximate relative luminosities of the two stars and a typical amplitude of the wave, Hall (1972) concluded that the fainter hemisphere of the cool star is typically about 2/3 as bright as the brighter one. The extent of the active region in longitude was taken to be 180° because the wave was very close to sinusoidal in most of the light curves considered. The model restricted the spot activity to about 30° on either side of the equator in analogy with the Sun. In this case the spotted region would occupy about 60% of the disk of the fainter hemisphere and the average temperature of the spotted region would need to be only about 1000 K cooler in order to render the fainter hemisphere to be 2/3 as bright as the brighter hemisphere. Hall (1972) noted that since the eclipses cannot be too far from central and since the radius of the hot star is less than half that of the cool star, it is predominantly a portion of the dark region, near the equator of the cool star, which is covered up at secondary minimum. The conclusion was that secondary minimum should indeed be too shallow by a factor of approximately two as observed.

Hall (1972) noted that it is generally thought that in relatively close binaries both components are rotating in synchronization with the orbital motion. According to the results of Popper in 1970, the line widths suggested that the components of RS CVn are rotating synchronously, at least approximately. Nonsychronization had been sometimes observed in Algol–like binaries. It was considered to be a temporary phenomenon driven by the infall of matter coming over from the companion star. Synchronization to the orbital motion would occur soon after the companion star ceased to eject matter.

In RS CVn there is likely to be no stream of matter flowing from one star to the other. Both components are well within their Roche lobes. Popper had emphasized in 1970 that the emission is quite different from the ring–type emission associated with the Algol–like binaries. This gave reason to expect synchronous rotation in RS CVn.

Hall (1972) gave also reason to expect that the individual components of RS CVn are rotating differentially. He considered differential rotation as a permanent equilibrium situation in stars having an outer convective envelope. It seemed to be a reasonable suggestion, providing only that the tidal bulge raised by the binary companion does not entirely suppress the natural tendency to rotate differentially. As a consequence the concept of strict synchronous rotation had to be abandoned, because a star rotating with different angular velocities at different latitudes simply cannot keep all parts of one side perpetually facing an orbiting companion. Hall (1972) noted that since observational rotation period, when suitably averaged, coincides with the orbital period, it follows that there is a particular latitude, somewhere between the pole and equator, which does rotate synchronously. Hall (1972) called this as the "corotating latitude."

Hall (1972) explained the migration of the wave in the direction of decreasing orbital phase by noting that the cool star (K0 IV), responsible for the migrating wave, is cool enough to have deep convective envelope. The equator could rotate slightly faster than the corotating latitude. This is analogous to the case of the Sun in which the equator rotates somewhat faster than higher latitudes.

According to Hall (1972), Catalano and Rodono had explained in 1969 the correlation between the depth of the primary eclipse with the migration of the wave. When the dark hemisphere of the cool star faces Earth at the primary minimum, the primary eclipse is unusually deep. When the bright hemisphere faces Earth, the primary eclipse is unusually shallow.

The observed displacement of secondary eclipse is a further consequence of the migration of the dark region, as Catalano and Rodono had noted in 1967. Whenever the dark region is facing either towards or away from the Earth at midsecondary minimum, then at conjunction the distribution of surface brightness on the cool star is symmetrical from one side of the disk to the other and minimum light occurs at conjunction. When the dark region faces the Earth at either quadrature, then at conjunction the dark region lies on one side of the cool star's disk or the other and minimum light occurs somewhat before or after conjunction. Catalano and Rodono have shown that the displacements are indeed in the proper sense.

Hall (1972) remarked that if there is a spot cycle operating in the cool star of RS CVn in analogy to the sunspot cycle, then the number (and/or size and/or darkness) of spots on its surface should vary more or less periodically. Consequently the brightness of the darker hemisphere relative to the brighter hemisphere should vary accordingly. Hall measured simply the difference between the maximum of the wave and the minimum to get estimates of the amplitude of the wave from all available observations. By plotting the amplitudes versus time it could be seen that they are represented by an approximately sinusoidal variation with a period of about 1800 orbital cycles or about 23.5 years.

Hall (1972) considered that this 23.5–year spot cycle makes it possible to understand why the wave migrates nonuniformly. In our Sun the average latitude of the spots depends on the phase of the sunspot cycle, being greatest at the beginning of the cycle and decreasing steadily towards the end. This gives rise to the familiar butterfly diagram.

Hall (1972) noted that if this phenomenon of spot drift occurs also in the cool star of RS CVn, then one would expect the migration of the dark region to be slowest just after each spot minimum, because the region of greatest activity will then be closest to the corotating latitude. Conversely, just before each spot minimum, the migration rate will be fastest because the region of greatest spot activity will then be farthest from the corotating latitude. It was realized by Catalano and Rodono in 1967 that the wave does not migrate at a uniform rate. But, Hall (1972) could infer from his migration curve for the minimum of the wave an average rate of 725 orbital cycles or 9.5 years per migration in RS CVn. Residuals from this average migration rate could be caused by latitudinal drift of spot activity. The maximum rate of 600 orbital cycles per migration occured just before starspot minimum and a minimum rate of 900 orbital cycles per migration just after starspot minimum. These discontinuities in migration rate occured at the epochs of spot minimum.

3.3 Asymmetries in absorption features

Fekel reported in 1980 high–dispersion spectroscopic observations of V711 Tau which showed that all absorption features of the active subgiant component were asymmetrical. Changes in these asymmetries were correlated with photometric phase (Fekel 1983). It occured to Vogt and Penrod that these distortions were essentially one–dimensional maps of the surface brightness of the rotating star. They considered that if enough phases are suitably combined, and the inclination of the star is known, a resolved image of the spotted star is derivable. Vogt and Penrod (1983) discussed more thoroughly the Doppler Imaging method and its preliminary application to the spotted star V711 Tau.

According to Rice (1996) the first significant efforts to map surface anomalies on stars goes back to the work of Deutch in 1958. He attempted to reconstruct the surface distribution of elements and the magnetic field on an Ap star.

The common procedure for surface imaging today is to invert the profile information to a map, using a minimization technique. Due to noise in the data the solution with minimum deviations is too unstable and the map is very noisy. There are a lot of stable and smooth solutions which are compatible with the data within a given level of noise. To obtain a unique appropriate solution, some additional constraints are necessary, and at present the surface imaging techniques differ mostly by the applied constraints. The Tikhonov method searches for the solution with the minimum gradient of the parameters across the map. The maximum entropy method selects the image with the minimum of information. Such kinds of smoothing methods are artificial and lead to an apparently acceptable, but distorted solution. This distortion, however, is reduced with increase of the quality of observations (Berdyugina 1998).

Recently a new approach to the inverse problem has been elaborated. The main advantage of this Occamian approach is that it does not use any artificial constraints to the solution except its non–negativity, which according to Berdyugina (1998) appears quite natural for a lot of inverse problems. Another advantage of the Occamian approach is its ability to analyse the available information and use it for estimating the errors of solution (Berdyugina 1998).

Strassmeier (1996) has reviewed the results of various surface imaging techniques applied to about thirty active late–type stars. It was found that the images usually show large cool features at different positions: a polar cap, high latitude spots, and low latitude spots. It was noted that polar caps and high latitude spots are long–lived structures, while low latitude spots are unstable on a short timescale. Due to different inversion methods and inputs for the line profile calculation, a comparison of the images even for a given star appeared to be very difficult.

3.4 Spectroscopic evidence for stellar analogues of sunspots

Horace W. Babcock first detected a magnetic field in the Ap star 78 Vir in 1946 (Preston 1967). Borra et al. (1982) mention that since this discovery magnetic fields have been reported in perhaps 200 stars, mostly with Ap spectra. Large scale magnetic fields, of strength from about 10^2 to 10^4 G, are observed at the surfaces of chemically peculiar stars (Ap and Bp stars).

The magnetic fields in these chemically peculiar stars vary strictly periodically, with magnetic and rotation periods equal. The magnetic fields on chemically peculiar stars are ordered on a global scale, without substantial smaller scale components. In almost all cases the field geometry can be well approximated by a dipole, often with its centre displaced from the centre of the star along the axis of field symmetry. There are only null field measurements for the Be stars, giving an upper limit to the poloidal field strength of about 100 G. However, in these stars the regular modulation of spectral lines formed in the wind regions at the rotation period is interpreted as indirect evidence for the presence of large scale fields, perhaps no more than of order 10 G, corotating with the star. Significant toroidal fields could also be present, but evade detection (Moss 1994).

Pallavicini (1988) reviews the Robinson method which gave the first positive detection of magnetic field of the order of 2 kG on 10–40 % of the stellar surface in two cool stars ξ Boo A and 70 Oph A. High resolution spectra with high signal to noise ratio have since been succesfully modeled to yield information on the magnetic properties of cool stars. These analyses yield estimates of the intensity weighted surface filling factor of active regions, f, and the mean unsigned field strength in these regions, B. The measurements are difficult, though, hampered by the small magnitude of the Zeeman effect, small f values for most stars, and the unknown spatial and thermodynamic properties of the active regions. These measurements in visible light refer to magnetic fields outside spot regions; since spots are much darker than the surrounding photosphere in visible light, their contribution to the integrated line profiles is expected to be neglible in typical stellar conditions. Recent data, mostly in the infrared where the Zeeman effect is larger, are yielding better measurements than ever before. Saar (1996) reviewed the new observations, and showed how they are expanding and modifying our understanding of magnetic fields on cool stars. Saar et al. modeled in 1990 Ti I flux profiles with a simple two component (magnetic/quiet) model. Ti I profiles in spectra of LQ Hya (dK2) gave $B \approx 3.5$ kG and $f \approx 0.7$. The equivalent width of the 2.2233 μm feature seemed to indicate a significant, cool, magnetic component. This represents the first direct measurement of magnetic field in stellar spots.

According to Saar (1996) there is very few reliable measurements of B and f in RS CVn stars. Zeeman Doppler imaging of V711 Tau measured only the net flux within spatial resolution elements; interpretation in terms of B and f values is not straightforward. Measurement of $B \approx 3$kG and $f \approx 0.60$ by Saar et al. in 1995 on II Peg represents one of the first such measurements for an active, evolved star. The Ti I W_λ values were too large to arise from the stellar photosphere. The infrared measurement had to come primarily from dark spots.

According to Neff et al. (1995) the idea that the TiO bands could be used to measure starspot properties was first stated by Vogt in 1979 and by Ramsay and Nations in 1980. From energy distributions over the wavelength range 5000–9000 Å obtained at phases when the spot (suggested by photometry) was both completely in and out of view the relative energy distribution of the spot was derived for II Peg by Vogt (1979). The spot's spectrum showed a steep rise in flux to the red and pronounced molecular absorption features of VO and the γ system of TiO characteristic of late–type stars. From the relative strengths and overall appearance of the molecular features, an estimate of $\geq$ M6 was obtained for the spot's spectral type, in good agreement of photometrically determined temperature (Vogt 1979, 1981). Ramsey and Nations (1980) obtained some high–dispersion spectrograms of V711 Tau. The TiO band system near 8860 Å was shown to strengthen greatly at phases when starspots were predicted according to photometry to be present on the visible hemisphere the G5 IV + K1 IV system V711 Tau. The 8860 Å transition is not produced in dwarfs warmer than $T_S \gtrsim 3500$ K (i.e. not in stars earlier than spectral type M2). So the absorption could not arise in the "normal" photosphere of either star (Neff et al. 1995).

Neff et al. (1995) described a technique using the absorption bands of the titanium oxide molecule at 7055 and 8860 Å to measure the effective temperatures and area coverages of starspots on active stars. They use the spectra of inactive M stars to model the spotted regions of active star photospheres and spectra of inactive G and K stars to model the unspotted regions. These proxy spectra are weighted by their relative continuum fluxes and by a surface area covering factor to reproduce spectra of active star. When these two TiO bands are measured in the spectra of active stars, the ratio of their depths is a function of the temperature of the starspots, and their absolute depths are functions of the total area of starspots on the visible hemisphere.

3.5 The active longitudes of FK Comae Berenices

3.5.1 The flip–flop of activity on FK Comae Berenices

The flip flop of the photometric minimum was discovered from the photometry of FK Com spanning over nearly a quarter of a century. Jetsu et al. (1991) reported the main results. The active regions of FK Com were shown to perform a flip–flop behaviour, i.e. the concentrated part of spot–activity shifts exactly to the other side of stellar surface, and then remains on the same longitude for a time interval from a few years to a decade. The activity showed excellent phase coherence with respect to these two active longitudes separated 180° from each other. The amount of differential rotation seemed very strictly limited, since the whole collected photometry could be described by one unique photometric rotation period with very small drifts of the photometric minimum.

Jetsu et al. (1993) published more detailed description of the observations and the results derived from them. The collected photometry of FK Com was divided into subsets with an average length of one month, since the light curve does not evolve significantly during this short time interval (Jetsu et al. 1991). The symmetric shape of these light curves resemble a sinusoid, although more detailed shapes have been observed in a few cases. The light curve is adequately presented by a second order Fourier fit (Jetsu et al. 1993).

The three stage period analysis of all normalized photometry gave the period $2.400226\ \mathrm{d} \pm 0.000015\ \mathrm{d}$. The phase coherence achieved with this value was far from satisfactory. This period implied that significant phase shifts were present. The photometry was then divided into four separate groups and the best periods were determined separately for each of these groups. Accuracy of these periods implied that the phase shifts of half a rotation were real (Jetsu et al. 1993). The modified normalized magnitudes were created to take maximum advantage from the otherwise disastrous effect of the phase shifts of half a rotation to the period analysis. New zero point of the ephemeris and period $2.4002466\ \mathrm{d} \pm 0.0000056\ \mathrm{d}$ were derived from the modified normalized magnitudes (Jetsu et al. 1993).

The ephemeris for the flip–flop of FK Com was derived assuming this new period to be constant. This assumption is equivalent to a model in which differential rotation combined with the latitudinal drift of the active regions can not be significant.

Phase coherence of the light minima of separate subsets were studied with bootstrap method. The weighted mean of all minima at phases $0.027\ \mathrm{d} \pm 0.046\ \mathrm{d}$ and $0.535\ \mathrm{d} \pm 0.071\ \mathrm{d}$ agree with the flip–flop definition.

The model of a constant photometric rotation period with the flip–flop gives excellent phase coherence for FK Com. The spot activity has remained concentrated on two active longitudes 180° apart from each other. The change of the active longitude can be rapid, since there was a gap of only about 190 d during which one flip–flop event occured. The activity is concentrated on one longitude at the time, because none of the normalized subsets showed a statistically significant secondary minimum (Jetsu et al. 1993).

Further UBVR photometry of FK Com produced two new subsets of data. These subsets showed that the concentrated part of the spot distribution was shifted to a new active longitude, which was consistent with the flip–flop ephemeris for a quarter of a century of photometry. The flip–flop ephemeris may predict where the activity shifts, but not when this event occurs (Jetsu et al. 1994b).

An alternative model which allows possible differential rotation combined with the latitudinal drift of the active areas was constructed. The unique phase–longitude relation is lost in this model, if significantly different values of the photometric rotation period are derived for the separate subsets of normalized photometry. It was concluded that measurable differential rotation is present for a model of varying photometric period (Jetsu et al. 1993).

Jetsu et al. (1993) asked: why is phase coherence with flip–flop achieved, when measurable differential rotation is certainly present? An additional parameter, the upper limit of the possible phase shift inside the subset, the coherence length was derived to clarify this contradiction. The coherence length is the upper limit of the possible phase shift inside the subset, if the photometric phase was derived either with the period of ephemeris or with the period of the subset.

The main result was that the varying and constant period models can be simultaneously valid. The spots are concentrated on the flip–flop longitudes even if the period is variable. The spots seem to form on these longitudes, but do not live long enough to experience a significant drift due to the differential rotation. In this scheme the longitudes of the flip–flop are long–lived, while the spots are short–lived and new spots would be continuously forming at the currently active longitude (Jetsu et al. 1993).

The spots may drift about ± 0.1 away in phase from the active longitudes of FK Com. The upper limit of differential rotation gave an approximate lower limit for the lifetimes of spots to be about eight rotations or 20 d. In conclusion the active longitudes could be a long–lived structure of the magnetic fields perhaps deeper in the photosphere. New spots would continuously form on the currently active longitude. The latitude of the formation of the majority of the spots would determine the observed photometric rotation period. The spots may fade even in 20 d before drifting far from the active longitudes. This tends to support the empirical experience that continuous light curves were achieved for subsets with an average length of one month (Jetsu et al. 1993).

3.5.2 Some surface imaging results

Surface imaging results of Korhonen et al. (1999) indicate that in the polar regions of FK Com higher latitudes rotate faster than the lower ones. This inverse sense of surface differential rotation compared to the Sun is not uncommon in rapidly rotating stars. More images are needed to confirm such rotation and to sample even lower latitudes in case spots are detected also there.

Korhonen et al. (1999) also combined all available photometry of FK Com in order to consider the phases of photometric minima. They found that the minimum is moving between the two active longitudes found by Jetsu et al. (1994b). The occupation of a given active longitude lasts about 1–2 yr, and moving to the other longitude also takes about 1–2 yr.

3.5.3 The concentrated flare activity on FK Comae Berenices

Jetsu et al. (1993) remarked that some authors had noted that the photometric flares of FK Com might tend to occur at preferred phases. Jetsu et al. (1993) noted also that because there is a unique phase–longitude relation for the model of a constant photometric rotation period, the location or time dependence of the flares of FK Com can be studied. An observed flare may originate from any longitude of the visible stellar surface, i.e. the uncertainty of the difference between the actual flaring longitude and the longitude in the centre of the visible disk is 0.25 in phase. This uncertainty would be even larger for flares above the photosphere. White light flares are assumed to occur near the stellar surface, as for example in the model reviewed by Haisch et al. (1991).

Jetsu et al. (1993) found from their photometric data that taking the flip–flop activity of FK Com in consideration flares seemed to occur preferably outside the longitude of the concentrated spot activity i.e. photometric minimum. Even the additional uncertainty of the actual flaring longitude did not seem to disturb this distribution, and thus it seemed likely that the photometric flares originate close to the stellar surface.

Foing (1989) notes that the studies of flare recurrence either on short time–scales or on longer time–scales (flaring active longitudes), may provide some clues on the small–scale and large–scale distribution of magnetic field on the stellar surface. Previous discussions reviewed by Pettersen (1989) include arguments for and against a random distribution of stellar flares in time. According to Jetsu et al. (1993) the study of UV Cet star EV Lac by Doyle (1987) is one of the few cases comparable to FK Com. Jetsu et al. (1993) noted that the solar analogy for FK Com could be the phase coherent active longitudinal zones of flares, except that the flares of FK Com would be located on the longitude 180° away from the concentration of spot activity. In the flip–flop scheme the long–term spot activity is also localized on the same active longitudes.

3.6 Active longitudes on some RS CVn stars

3.6.1 Some photometric results

Henry et al. (1995) found some indications that rigid quadrant structure was manifested in all their two–spot modeled stars λ And, σ Gem, II Peg, and V711 Tau. Evidence was more convincing in some, less in others. They listed the spots which appeared in the vicinity of one of the two supposed active quadrants for each star. They calculated parameters characterizing the two active quadrants from the first appearance of the spots on the longitudes. These parameters were the total widths of the two active quadrants, the longitude of the center of the two active quadrants and the longitude difference between those two centers. They considered the strength of the quadrant structure in these stars by two measures, namely by widths of the active quadrants, which should be preferentially less than 0.25 orbital phase units or 90°, and that the two active quadrants should be 0.5 phase units or 180° apart. Width of active longitudes varied from 0.38 phase units in case of V711 Tau to 0.18 in case of σ Gem. So σ Gem was considered to posses the most convincing active quadrants by these measures. The centers of active longitudes were approximately at opposite hemispheres from each other in all the modeled stars, which made them all fairly strong in this measure (Henry et al. 1995).

The preferred longitudes will be the longitudes of the two active quadrants in each spotted star which occur stationary in a synchronously rotating binary. Preferred longitudes suspected in previously investigated systems have not consistently favored any one particular alignment (e.g. Zeilik et al. 1987; Zeilik 1991). Henry et al. (1995) met also an inconsistency in their results. They found two active quadrants in σ Gem to be fairly close to the two conjunctions. The two active quadrants in II Peg and the two in V711 Tau were found to be fairly close to the two quadratures. Because λ And is an asynchronous rotator, no preferred longitudes could be found (Henry et al. 1995).

Nonparametric methods to search for periodicity in a weighted and nonweighted time point series were applied by Jetsu (1996) to the central meridian passage epochs of starspots determined by Henry et al. (1995). Active longitudes could be detected uniquely, regardless of the presence of differential rotation or abrupt shifts in the longitude of main activity. According to results of Jetsu (1996) three of these binaries most probably have only one asynchronously rotating active longitude: λ And, II Peg, V711 Tau. The case of σ Gem was different, because two stable synchronously rotating active longitudes separated by 180° were clearly present, and the temporal changes connected to these centres of activity resembled the flip flop phenomenon detected earlier in FK Com (Jetsu 1996).

The same photometric data was also reanalyzed for σ Gem and for II Peg by Berdyugina and Tuominen (1998). The original light curves were analyzed with the suggestion that the deepest minimum in the light curve is caused by the largest active region on the stellar surface consisting probably of a group of spots. The secondary minimum, if present, should correspond to another, smaller active region. Then, tracing the minima in time could provide information on migration of the active regions with no assumptions on their structures, shapes, areas, temperatures, and latitudes.

3.6.2 Some surface imaging results

Berdyugina et al. (1998a) reported nine new images of II Peg for 1992–1996. No evidence for polar cap was found. Persistent high–latitude spots were restored, which are responsible for spectroscopic and photometric variability of the star.

The Occamian approach, which has recently been applied to surface imaging, showed also that errors in the maps increase significantly towards the lower latitudes (Berdyugina 1998). Therefore, the restored high–latitude spots appear to be more reliable than the low–latitude spots, especially near– and sub–equatorial features. Spots are often grouped in two active regions separated by half a period in average. The two active regions have survived during 4.5 years in the images and can be interpreted as a long–lived active longitude structure on the surface of II Peg. The new analysis of the published photometric data confirms that these permanent active longitudes have existed during at least 23 years (Berdyugina and Tuominen 1998). In the images one of the active regions is generally larger than the other one, which determines a quasi–sinusoidal shape of the light curve with one minimum. In 1994 a formerly larger active region has decreased its area, while the other one has increased and survived as a larger one up to fall 1996. In other words, this has indicated switching the activity between the longitudes. From the photometric data it was also found that such switching happened several times during 23 years in the same period of time, about 4.65 years. A double value, 9.3 year, can be regarded as a total cycle length during which both longitudes are subsequently active. With such a definition of the stellar activity cycle one can try to predict in advance the next event of switching the active longitudes. New observations which have been carried out in 1997–1998 have provided four new images of the star. The alteration of the spot activity revealed in July 1998 corresponds to a new event of switching the active longitudes, similarly to the event of 1994 mentioned above. It has happened only half a year earlier than predicted from previous observations (Berdyugina 1999).

Two permanent, migrating active longitudes, separated by half a period, which have been discovered on the surface of II Peg with the surface imaging technique, have been also revealed on the surfaces of other RS CVn stars from the analysis of long-term photometric observations (Berdyugina and Tuominen 1998). Also, the effect of alteration of the active longitudes on II Peg was found to be common on other stars. II Peg with six observed flip–flop events is the most convincing case. RS CVn stars EI Eri, II Peg, HR 7275 and IM Peg seem all have active longitudes which have no preferable orientation with respect to the line of centres in the binary. So these stars are not synchronized or the latitudes in which spots preferentially lie are not synchronized with the orbital motion. Only in the case of σ Gem the active zones seem to be synchronized. All stars show similar behaviour: one longitude is usually more active than the other, and the changing of the activity between the longitudes seems to be cyclic with periods of a few years. Alteration of the activity happens on a much shorter time scale, during a few months. The almost linear phase migration of the active regions suggests a very small, if at all, latitude motion of the spots (Berdyugina 1999).

Berdyugina (1999) emphazises the fact that FK Com has same kind of active longitudes as these RS CVn stars. Only difference seems to be the fact that for II Peg the largest active region is seen in the images close to the central meridian. For the chosen ephemeris it means that the largest active region tends to be faced towards the secondary. The presence of the secondary apparently affects somehow the activity of the primary. But, binarity is not required, since FK Com seems to be a single star. It seems that rapid rotation is sufficient condition to produce active longitudes and same kind of cyclic activity on these stars. Despite the fact that FK Com as a possibly coalesced binary may differ structurally very much from these RS CVn binaries.

Chapter 4

Theory

4.1 Basic magnetohydrodynamic equations

Some dynamic effects arising in the electrically conducting material, such as solar plasma, can be analysed with the help of Navier-Stokes and induction equations describing the evolution of the velocity and magnetic fields respectively.

The Navier-Stokes equation, or the equation of linear momentum conservation, for compressible flow is

$$\varrho \left[\frac{\partial}{\partial t}\mathbf{u} + (\mathbf{u} \cdot \nabla)\,\mathbf{u} \right] = \varrho \mathbf{g} - \nabla P + \nabla \cdot (\varrho \pi) + \mathbf{j} \times \mathbf{B}, \tag{4.1}$$

where ϱ is the density, $\mathbf{u}$ velocity, $\mathbf{g}$ acceleration due to gravity, P pressure and π viscosity tensor. The Lorentz force term $\mathbf{j} \times \mathbf{B}$ is the product of electric current density $\mathbf{j}$ and magnetic field $\mathbf{B}$. In the viscosity tensor

$$\pi_{ij} = \nu\,(u_{i,j} + u_{j,i}) + (\zeta - 2\nu/3)\,u_{m,m}\delta_{ij} \tag{4.2}$$

ν is the kinematic shear viscosity and ζ the kinematic bulk viscosity. Usually the analysis is simplified by considering the fluid to be incompressible ($\nabla \cdot \mathbf{u} = \mathbf{0}$) so that the right hand term containing $u_{m,m}$ can be omitted. For detailed discussion refer to Shore (1992), who derives the Navier-Stokes equation by extending the equations of motion by the introduction of long-range coupling within the fluid caused by viscosity.

Interaction of electrically conducting matter with a magnetic field in the non-relativistic case can be described with the induction equation

$$\frac{\partial}{\partial t}\mathbf{B} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B}), \tag{4.3}$$

where $\mathbf{B}$ is the magnetic field, $\mathbf{u}$ is velocity field and η is magnetic diffusivity. Here we refer the reader to e.g. Stix (1989) to see how the induction equation can be derived from the Maxwell's equations.

4.2 Ideal fluid and magnetohydrodynamic approximation

From the Navier-Stokes equation (4.1) a dimensionless number, called the Reynolds number, can be constructed

$$\mathrm{Re} = \frac{\mathrm{UL}}{\nu}, \tag{4.4}$$

where U is velocity, L is any typical scale of length in the system and ν is the kinematic viscosity. This parameter is named after O. Reynolds, whose investigations of the transition to turbulence were the pioneering effort to understand the role of viscosity in the phenomenon. It describes the rate of transport of momentum by viscosity versus advection (Shore 1992). The Reynolds number can also be written as a fraction of two time scales $\mathrm{Re} = \tau_{\mathrm{D}}/\tau_{\mathrm{A}}$, where $\tau_{\mathrm{D}} = L^2/\nu$ is the time scale of decay and $\tau_{\mathrm{A}} = L/U$ is advection time scale (Rüdiger 1989). Another dimensionless number, namely the magnetic Reynolds number $\mathrm{R_m} = \mathrm{UL}/\eta$, where η is called the magnetic diffusivity can be constructed from the induction equation (4.3). $\mathrm{R_m}$ can be understood as the ratio of the time scale of ohmic decay and the advection time scale (Stix 1989).

The conducting fluid in stars and other cosmical objects can be treated as an ensemble object or system with the help of some carefully chosen scales of length and time. At some magnification of scale or some rate of clock ticking, one can always apply a fluid approximation to the problems at hand. Viscosity dominates over a length scale L when the Reynolds number is very small. But, there is always some length scale over which the viscosity will be important, provided the fluid is not ideal, and therefore there is some minimal length below which, for any arbitrary velocity U, the ideal fluid approximation does not apply (Shore 1992). Shore (1992) notes that: "Since a star is composed of gas which is homogenous (in most cases) and which acts collectively to create its own gravitational field, it mimics rather well the behaviour of a fluid moving (or sitting) under gravity. The collision times are so short (or, put another way, the mean free paths are so short compared with any scale lengths in the medium) in the interior of the star that any disturbances can be washed out and the structure can be described as continuous."

Everywhere in the stellar atmospheres a sufficient number of free electrons are available so that electric current can flow. This demands the consideration of various electromagnetic effects. In combination with the neglet of the displacement current in Ampère's law (4.5), and the nonrelativistic form of Ohm's law (4.6), this constitutes the magnetohydrodynamic approximation (Stix 1989)

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}, \tag{4.5}$$

$$\mathbf{j} = \sigma \left(\mathbf{E} + \mathbf{u} \times \mathbf{B}\right). \tag{4.6}$$

In the equations above $\mathbf{B}$ is the magnetic field, μ_0 is magnetic permeability of free space, $\mathbf{j}$ is electric current density, σ is electric conductivity, $\mathbf{E}$ is electric field and

u is velocity. Ampère's law is the key to the connection between hydrodynamical and magnetic phenomena. The current behaves like the fluid mass current of the normal mechanical equations. The current is carried by one of the components of the medium, specifically the electrons, and they are strongly interacting with the protons by collisions. It can be assumed that the medium is completely neutral (i.e. in strict charge balance) to simplify treatment and that ionization balance occurs in the plasma. The presence of an electric field across the medium causes a current to flow, but it is assumed that the collision time between electrons and ions is very short and thus the collisions act to produce a bulk motion of the ions and electrons. The displacement current in Ampère's law can be negleted, and the bulk variables of density, pressure, and mean velocity can be used to describe the plasma. Provided that the plasma cannot support any internal electric fields, the current is determined only by the magnetic field, and the charges do not separate, then one is justified in choosing a fluid description of plasma (Shore 1992).

4.3 Turbulence

Shore (1992) describes the studies of Reynolds which pointed to the action of viscosity as a agent for the onset of turbulence: "Reynolds" hydrodynamic studies, concluded in the second half of the nineteenth century, were in the connection of channel flow. He found that dye tracers moved smoothly in a fluid flowing in a flat, narrow laboratory channel when the characteristic number now named after him was small. That is, laminar flow is a characteristic of small Reynolds number, when viscosity dominates. However, the flow underwent a striking transition for large values of the number. The dye marking the channel flow became turbid and mixed rapidly. There was no evidence of a precursor stage to this change. It had an onset very much like a phase transition. Reynolds argued that the causative agent was the generation of vorticity by shear in the fluid which on some scale was causing dissipation and that the onset of turbulence must be assosiated with the action of viscosity."

In 1895 Reynolds pointed to the existence of additional, turbulently generated stresses which are basic to many areas of fluid dynamics. Boussinesq in 1897 and others later related the Reynolds stresses to the gradients of the mean flow. It was noted that quotients of turbulent fluxes and mean gradients yielded consistent values. Using modern notation this idea is expressed mathematically

$$Q_{ij} = \overline{u_i{}' u_j{}'} = -\nu_{\mathrm{T}} \left(\bar{u}_{i,j} + \bar{u}_{j,i} \right), \tag{4.7}$$

where $Q_{ij} = \overline{u_i{}' u_j{}'}$ is the correlation tensor (Rüdiger 1989). Reynolds stresses in this tensor form resemble the viscosity tensor (4.2). The main difference is that the velocities are replaced by their respective mean quantities and the molecular viscosity ν is replaced by the turbulent viscosity ν_{T}. The Austausch A (i.e. the coefficient relating stress to rate-of-strain) become calculable in 1925 when Prandtl introduced the concept of the mixing length (Rüdiger 1989). By 1932 the Austausch was already known to be the product of the density, the mixing length and the root mean square (r.m.s.) velocity of the turbulent element i.e.

$$\nu_{\mathrm{T}} = A/\rho \approx Lu' \approx L^2/\tau_c, \tag{4.8}$$

where τ_c is the lifetime, or turnover time, of the element. (Rüdiger 1989). Order-of-magnitude estimate of $\mathcal{Q}_{ij} = \overline{u_i' u_j'}$ shows that $\nu_{\mathrm{T}} = lu$, where l and u are the typical scale and speed of the velocity field $\mathbf{u}$ (Stix 1989). If the scales of granules ($l \approx 100000000$ cm and $u \approx 100000$ cm/s) are combined together as an "eddy viscosity" a value of 10^{13} cm^2/s is obtained for ν_{T} (Rüdiger 1989). It can be concluded that the eddy diffusitivity is larger than molecular diffusitivity by many orders of magnitude in the solar convection zone (Stix 1989).

The importance of the enhancement introduced by turbulence to some quantities becomes apparent when one considers e.g. the solar magnetic cycle and the equatorial acceleration of solar rotation. Rüdiger (1989) points to the possibility that the solar viscosity may be much larger than the molecular viscosity: "If neither meridional circulation nor magnetic forces are present, any large-scale non-uniform rotation is always smoothed out in a characteristic time of $\tau_{dec} \approx \mathrm{R}^2/\nu$. When the molecular viscosity and solar radius are used, τ_{dec} is very large: $\tau_{dec} \approx 50 \cdot 10^{20} \cdot 10^{-3}$ s $\approx 10^{11}$ yr. Were this the whole story, no special explanation of differential rotation would be called for, since it could be thought of as a fossil from the birth of the Sun. The fact, however, that the northern and southern hemispheres have identical rotation laws already makes the fossil hypothesis seem unlikely. Moreover, no other long-lived, hydrodynamical phenomenon has been observed on the Sun. The 'true' viscosity may therefore be much larger than the molecular viscosity." Indeed, by using the "eddy viscosity" instead of molecular viscosity one gets the estimated decay timescale for differential rotation in turbulent enviroment $\tau_{dec} \approx 50 \cdot 10^{20} \cdot 10^{-13}$ s ≈ 16 yr. Also comparing decay times for global stellar magnetic fields using molecular and "eddy" magnetic diffusitivity leads to similar figures (Roberts 1994) :"... if we use a magnetic diffusitivity based on the probable electrical conductivity of the solar plasma, we find that electromagnetic diffusion time is order of 10^{12} yr. ... And it certainly seems to be true that the Sun is able to purge its surface magnetic field every decade or so, and to then create a new magnetic field of opposite polarity that lasts for a similar duration. Many stars of solar type exhibit similar cycles, suggesting again that their effective magnetic decay times are measured in decades rather than trillions of years."

The turbulence need not always be a cause of destruction. It can give rise to large-scale structures under certain conditions. Next the development of ideas how the solar magnetic cycle and solar differential rotation can be explained are reviewed.

4.3.1 Discovery of Λ–effect

The following discussion of the discovery of Λ–effect is largely based on Rüdiger (1989):

Frederick Zöllner was particularly impressed by the rotation of celestial bodies and saw it "as the single obvious reason for the latitude–dependence of physical quantities. Rotation produces a streaming in the outer layers of a centrally heated liquid sphere, resulting in a negative pole–equator temperature difference." He published in 1881 an explanation how a meridional flow could affect rotation. He argued that the flow towards the equator of a thin layer of fluid on the surface of a rigidly rotating sphere would produce a frictional force on the sphere's surface. Under several preconditions Zöllner calculated the increase in the zonal velocity of the liquid. He claimed that application of his formula yielded a better account of the observations of solar differential rotation than the relation given by Carrington. From the existence of differential rotation, Zöllner thoughtfully inferred the "rigid state of sunspots", which otherwise would be sheared into spiral stripes.

Wilsing in 1891 in Potsdam argued that if the viscosity is sufficiently small in the outer parts of the Sun, the viscous decay time corresponding to its large dimension will be so long that the circulation may be considered as a fossil flow. Emden argued against the relic theory in 1907: "The constancy of the Sun's effective temperature requires convective flows to mix the material. Cooled surface elements are replaced by new warmer elements. Any equatorial acceleration would be smoothed out by the mixing process . . . Only a steady source can maintain the equatorial acceleration against the action of eddies." Like Helmholtz he postulated the existence of eddies which were thought to emerge from certain discontinuity surfaces within the Sun. These eddies, which destroyed any regularity in the flow, also prevented solid–body rotation from occuring. Nothing could be said against the hypothesis of a fossil rotation profile without dramatic changes in the effective viscosity. In 1932 Ludwig Biermann introduced the concept of the mixing length into astrophysics: he was in close contact with Prandtl. Biermann discussed turbulently–induced enhancements of the viscosity and thermal conductivity by as much as a factor of a million.

More recent investigations confirm the findings of the pioneers: that stochastic motions take place in the Sun's outer layers, that these are large–scale motions, and that they do not necessarily have the same properties in all directions. On this basis A. J. Lebedinski searched reason for differential rotation. Lebedinski treated turbulence and Reynolds stresses as the dominant physical phenomena at and below the solar surface. Because of the enormous scales the "Austausch", must be necessarily anisotropic. He argued in 1940 that " Two coefficients should be distinguished, a radial one and a horizontal one (A_{V} and A_{H}). The non–isotropic character of turbulent friction is brought about because the mean values of the velocity and the mixing are different in different directions." Lebedinski demonstrated that rigid rotation is not the appropriate solution of the ϕ-component of the Reynolds equation, the conservation of angular momentum.

Lebedinski was the first to make the non–vanishing of the angular momentum transport for originally uniform rotation the basis of the theory of differential rotation. In this sense he may be regarded as the discoverer of the phenomenon which is today called the Λ–effect, where the symbol Λ reminds us of his contribution to Astrophysics.

In 1946 Wasiutyński suggested that different viscosities apply to the three principal directions of the spherical coordinates. He calculated the consequences of this assumption on the correlation tensor $\mathcal{Q}_{ij}$. The results contained already an essential difference from the Boussinesq relations: anisotropic turbulence transports angular momentum even when the mean rotation is rigid, i.e. $\Omega =$ constant no longer satisfies the angular momentum conservation law. Biermann in 1951 treated turbulent angular momentum transport in a way similar to Wasiutyński. Biermann was the first to find solar rotation law for the case of anisotropic turbulence. It depended, however, on depth and not latitude. Kippenhahn developed in 1963 an iteration procedure for high viscosity and/or slow rotation and found that turbulence alone gave rise to a radial rotation law, which then generated the required meridional circulation that, in turn, produced the acceleration of the equator. This approach gave a meridional flow velocity which is all too much high compared with solar circulation velocity. Later studies lowered the velocity to the level indicated by observations. Rüdiger (1989) showed that the velocity at surface may even vanish completely. In 1969 Iroshnikov developed an approximation for the correlation tensor $\mathcal{Q}_{ij}$ to higher orders in Ω, that is to faster rotations. Iroshnikov could derive from these Reynolds stresses a latitude dependent rotation law. He recognized that not only meridional circulation but also the Ω^3-effect (that is "fast" rotation) could produce an equatorial acceleration. Iroshnikov had opened up a new area of investigation. The Reynolds stresses could produce the observed differential rotation. It was now obvious that the correlation tensor in a rotating turbulence $\mathcal{Q}_{ij}$ had to be investigated as carefully as possible.

4.3.2 Discovery of α–effect

The idea that the magnetic field of a cosmical object may be of dynamo origin was presented first time, when Sir Joseph Larmor read his paper "How could a Rotating Body such as the Sun become a Magnet?" at the seventh meeting of the British Association for the Advancement of Science on September 9th, 1919. He analysed the question for the origin with the conclusion that "it is possible for the internal cyclic motions to act after the manner of the cycle of a self–exciting dynamo, and maintain a permanent magnetic field from insignificant beginnings, at the expense of some energy of the internal motions" (Krause 1993).

From this point began the era of study of systems cabable to the dynamo action maintaining the magnetic fields observed in the heavenly bodies. Moffatt (1978) noted that it was natural for early investigators to analyse systems having a maximum degree of symmetry in order to limit the analytical difficulties of the problem. The most natural primitive system to consider in the context of rotating bodies is one in which both the velocity field and magnetic field are axisymmetric (Moffatt 1978). Cowling proved in 1934 that a steady magnetic field being symmetric with respect to a given axis cannot be maintained by a steady velocity field which is symmetric with respect to the same axis. It can be shown that such a magnetic field cannot be maintained by any other velocity field either. This first "anti-dynamo" theorem was generalized and reinforced by later investigations (Krause and Rädler 1980).

It became clear that non-axisymmetric configurations had to be considered if any real progress in dynamo theory were to be made. Recognition of the three-dimensional nature of the problem led Elsasser in 1946 to initiate the study of the interaction of a prescribed non-axisymmetric velocity field with a general non-axisymmetric magnetic field in a conducting fluid contained within a rigid spherical boundary, the medium outside this boundary being assumed non-conducting. The pioneering study of Bullard and Gellman in 1954 showed clear recognition of the desirability of two ingredients in the velocity field for effective dynamo action: (i) a differential rotation which would draw out the lines of force of the poloidal[1] magnetic field to generate a toroidal field, and (ii) a non-axisymmetric motion capable of distorting a toroidal line of force by an upwelling followed by a twist in such a way as to provide a feedback to the poloidal field (Moffatt 1978).

Based on a concept suggested in 1961 by M. Steenbeck a rather general theory has been developed which describes the behaviour of mean electromagnetic fields, i.e. of the large–scale parts of the electromagnetic fields, in electrically conducting matter carrying out turbulent motions. This theory is generally called "mean–field electrodynamics". A completely disordered, i.e. homogenous isotropic mirror symmetric turbulence only influences the decay rate of the mean magnetic fields, which is enhanced in almost all cases of physical interest. A turbulence with the weakest possible deviation from complete disorder is one which is still homogenous and isotropic but lacks mirror symmetry. A homogenous isotropic non–mirror symmetric turbulence provides for the quite unusual effect of the appearance of a mean electromotive force parallel to the magnetic field, the so–called α–effect. In the convective layers of rotating bodies, the Coriolis force provides for a helical structure of the motions. In this case, right–handed and left–handed helical motions do not occur with the same probability on the same hemisphere. There are regions in which one kind of helical motions dominate, thus causing the α–effect (Krause and Rädler 1980).

[1] It should be noted here that the field can be decomposed to poloidal and toroidal parts. In the spherical geometry the poloidal field component is contained in the meridional plane. Toroidal field component is perpendicular to the meridional plane. The meridional plane is determined in the stellar case by rotation axis ($\hat{\mathbf{\Omega}}$) and unit vector $\hat{\mathbf{r}}$ defining the longitude of the plane.

4.4 Mean–field magnetohydrodynamics

In the equations (4.1) and (4.3) the effects of turbulence are not considered. Shore (1992) gives us clue how to proceed: " . . . we assume that the scale on which dissipation takes place is microscopic. To analyse the flows, however, does not really require any detailed knowledge of the underlying physical mechanism for viscosity. At the 'engineering' level of astrophysical fluids, we can turn the problem around and ask what viscosity we require to produce the observed flow. Ultimately, however, we have to ask where the viscosity comes from. Whatever we take as the dissipative mechanisms for the flow, they always act on scale smaller than we can directly observe. In this way, any chaotic, dissipative motions can play the same role as viscosity, on some length or timescale. This means we have to change our magnification when viewing different parts of medium. In fact, we have to ask different questions depending on the scale. On some level, for some range of velocities and lengths, any fluid is inviscid. On some other, it is very likely it will be like molasses. Or more likely, on some scale the description of fluid motion may break down altogether and we will have to resort to a kinetic description."

One fluid "engineering" approach to solar turbulent magnetohydrodynamic problems is to use the Reynolds' averaging technique (see e.g. how Stanišić 1988; Krause and Rädler 1980 introduces the technique). The velocity field $\mathbf{u}$ and the magnetic field $\mathbf{B}$ can be written as a sum of mean and fluctuating parts

$$\mathbf{u} = \bar{\mathbf{u}} + \mathbf{u}', \qquad \mathbf{B} = \bar{\mathbf{B}} + \mathbf{B}'. \tag{4.9}$$

Above $\mathbf{u}$ is the total velocity field, $\bar{\mathbf{u}}$ is the mean velocity field and $\mathbf{u}'$ is the fluctuating part of velocity field. $\mathbf{B}$ is the total magnetic field, $\bar{\mathbf{B}}$ is the mean magnetic field, $\mathbf{B}'$ is the fluctuating part of magnetic field. The line over fields notifies an averaging operation resulting to a mean value. The mean value can be understood as an expectation value from an ensemble of identical systems. Instead of averaging over an ensemble, mean values can also be defined by integration over space and time (Krause and Rädler 1980).

By substituting equations (4.9) to equations (4.1) and (4.3) and separating the mean and fluctuating parts one gets the Reynolds equation (4.10) and dynamo equation (4.11) describing the evolution of mean fields

$$\varrho \left[\frac{\partial}{\partial t}\mathbf{u} + (\mathbf{u}\cdot\nabla)\,\mathbf{u} \right] = \varrho\mathbf{g} - \nabla P + \nabla\cdot(\varrho\pi) + \mathbf{j}\times\mathbf{B} - \nabla\cdot(\varrho\mathcal{Q} - \mathcal{B})\,, \tag{4.10}$$

$$\frac{\partial}{\partial t}\mathbf{B} = \nabla\times(\mathbf{u}\times\mathbf{B} - \eta\nabla\times\mathbf{B} + \mathcal{E})\,. \tag{4.11}$$

Above the mean fields are written with bold face without the line indicating averaging operation. This changed notation is applied through out the rest of the treatment. Refer to Stanišić (1988) and Stix (1989) for detailed derivations of these equations.

Noteworthy in these equations are the correlations describing the effects of turbulence: turbulent Reynolds stresses $\mathcal{Q}_{ij} = \overline{u_i' u_j'}$, turbulent Maxwell stresses $\mathcal{B}_{ij} = \overline{j_i' B_j'}$ and turbulent electromotive force $\mathcal{E}_i = \overline{(\mathbf{u}' \times \mathbf{B}')}_i$. The turbulent electromotive force can also be derived by using Reynolds averaging technique on Ohms law (4.6), which results to $\mathbf{j} = \sigma \left(\mathbf{E} + \mathbf{u} \times \mathbf{B} + \mathcal{E} \right)$ (Steenbeck and Krause 1969; Krause and Rädler 1980). Usually the viscous π_{ij} and Maxwellian stresses $\mathcal{B}_{ij}$ are omitted in studies involving the Reynolds equation (4.10). Whenever turbulent motions exists in a star, the viscous stresses π_{ij} are neglible compared with the Reynolds stresses $\mathcal{Q}_{ij}$ (Rüdiger 1989). The form of tensor $\mathcal{B}_{ij}$ is poorly known in the stellar convection zones, and, at least in the solar case, the magnetically induced flows appear to be less important than those due to turbulence (Barker 1993).

It is the main problem for the mean-field magnetohydrodynamics to find some suitable representation for these correlations so that the used system of equations can be integrated. A tensor representation is useful in decomposing these correlations to separate parts describing various effects arising in the turbulence. The needed tensor elements are obtained through geometrical considerations and taking into account various anisotropic effects such as stellar rotation and gravitation. It may be advantageous sometimes to work with the Fourier transform of correlation tensor i.e. with its spectral tensor (Moffatt 1978; Krause and Rädler 1980; Rüdiger 1989). The magnitudes and signs of the tensor elements and various constants obtained can be derived by simple analytical order of magnitude estimates or by relating the equations directly to astronomical observations. Other alternative is to simulate magnetoconvection with more detail in some appropriately restricted domain and examine various occuring effects (Brandenburg 1994).

Actually the determination of turbulent cross correlations gives rise to an infinite set of equations. To keep the system of equations finite some closure technique is needed. This is the well known general problem of turbulence theory, which is not yet satisfactorily solved. Usually the theories developed take into account the turbulent fields up to the second order only (the first-order-smoothing approximation). This means that the cross correlations in the dynamo and Reynolds equation can be determined with the help of linearized equations for the fluctuating fields. It can be shown that this is appropriate if the condition $\min \left(R_{\mathrm{m}}, S \right) \ll 1$ holds. $R_{\mathrm{m}} = u' \lambda_{\mathrm{cor}}$ is the magnetic Reynolds number and $S = u' \tau_{\mathrm{cor}} / \lambda_{\mathrm{cor}}$ Strouhal number. The correlation time τ_{cor} and the correlation length λ_{cor} are the two scales characterizing the variotion of $\mathbf{u}'$ with time and position. The notation u' is for the r.m.s. velocity fluctuation i.e. $u' = \sqrt{\overline{\mathbf{u}'^2}}$. The first-order-smoothing approximation is applicable in a certain sense with weak turbulence only. However, effects deduced in the frame of this theory can be very large without breaking up the scales (Krause and Rädler 1980).

4.4.1 Turbulent electromotive force

By considering the equations for mean and fluctuating magnetic fields it becomes apparent that there must be a linear relationship between the turbulent electromotive force and the mean magnetic field (Stix 1989). Hence it may be reasonably anticipated that the turbulent electromotive force may be developed as a rapidly convergent series of the form

$$\mathcal{E}_i = \overline{\mathbf{u}' \times \mathbf{B}'}_i = \alpha_{ij} B_j + \beta_{ijk} \frac{\partial B_j}{\partial x_k} + \dots, \tag{4.12}$$

where the coefficients α_{ij} and β_{ijk} are pseudotensors (Moffatt 1978). This is the most general form of the turbulent electromotive force, but it takes the particularly simple form in the case of "weakly" isotropic turbulence

$$\mathcal{E} = \overline{\mathbf{u}' \times \mathbf{B}'} = \alpha \mathbf{B} - \beta \nabla \times \mathbf{B}, \tag{4.13}$$

where $\alpha = -\frac{1}{3}\overline{\mathbf{u}' \nabla \times \mathbf{u}'}\tau$ and $\beta = \frac{1}{3}\overline{\mathbf{u'^2}}\tau$ are constant mean quantities determined by the turbulent velocity field $\mathbf{u}'$ (Steenbeck and Krause 1969). This is the case where the statistical properties of $\mathbf{u}'$ are invariant under rotations but not generally under reflexions of the frame of reference. This isotropic nonmirrorsymmetric turbulence is usually interpreted to be caused by the helical motions induced by stellar rotation i.e. Coriolis force. This calls for the latitude dependence of the α-effect with the changing sign at stellar equator usually adopted in the form $\alpha \sim \cos\theta$ first investigated by Steenbeck and Krause (1969).

With the help of equation (4.13) the dynamo equation (4.11) can then be written as

$$\frac{\partial}{\partial t}\mathbf{B} = \nabla \times (\mathbf{u} \times \mathbf{B} + \alpha \mathbf{B} - \eta_{\mathrm{T}} \nabla \times \mathbf{B}), \tag{4.14}$$

where $\eta_{\mathrm{T}} = \eta + \beta$ is the turbulent magnetic diffusivity. The term involving α ensures that the solution $\mathbf{B}$ is not subject to the antidynamo theorems. Quite simple dynamos can be conceived with the help of this α effect. In most cases β can be considered to be positive (Moffatt 1978). The order of magnitude estimate $\beta = \frac{1}{3}u^2\tau \gg \eta$ leads to the substantial increase of diffusivity from η to $\eta_{\mathrm{T}} = \eta + \beta$ (Stix 1989).

More generally in the turbulent electromotive force α may be anisotropic (e.g. Rüdiger and Kitchatinov 1993), β does not have to be positive (Moffatt 1978) and higher order terms in expansion (4.12) may also give some contribution to the turbulent electromotive force (Brandenburg 1994).

4.4.2 Turbulent Reynolds stresses

In a nonmagnetic stratified rotating turbulence there are always two preferred directions. Both rotation and gravity will modify the turbulence and, consequently, they should both appear in the correlation tensor. This is the reason why the first workers in this field felt that the old Boussinesq relation (4.7) was too simple to be applicable to astrophysics. The statistics are quite different from those of an ordinary gas. The mean free path and mean free time between collisions are much too short for molecular motions to be influenced noticeably by rotation or gravity, but the relatively long lifetime of the turbulent eddies invites the introduction of an anisotropic "Austausch" (exhange) (Rüdiger 1989).

In a completely general turbulence theory, the scalar viscosity ν is replaced by a eddy viscosity tensor. In the most general linear case, this tensor must be of the fourth order (Stanišić 1988). The correlation tensor $\mathcal{Q}_{ij}$ is linked to the rate of mean strain tensor $u_{k,l}$. with the eddy viscosity tensor $\mathcal{N}_{ijkl}$:

$$\mathcal{Q}_{ij} = -\mathcal{N}_{ijkl} u_{k,l}. \tag{4.15}$$

Tensor $\mathcal{N}_{ijkl}$ is symmetric in i and j by definition of $\mathcal{Q}_{ij}$, and in general it depends on rotation, $\mathbf{\Omega}$, and gravity, $\mathbf{g}$: $\mathcal{N} = \mathcal{N}(\mathbf{g}, \mathbf{\Omega})$ (Rüdiger 1989). Rüdiger (1989) constructed the viscosity tensor $\mathcal{N}_{ijkl}$ in the incompressible case using $\hat{\mathbf{g}}$ and $\delta_{i,j}$ and neglecting the dependence on $\mathbf{\Omega}$. As a result he got the correlation tensor

$$\mathcal{Q}_{ij} = -\mathcal{N}_{ijkl} u_{k,l} = \mathcal{Q}_{ij}^{(d)} + \mathcal{Q}_{ij}^{(r)}, \tag{4.16}$$

from which the "differential" $\mathcal{Q}_{ij}^{(d)} = -\nu_{\mathrm{T}}(u_{i,j} + u_{j,i})$ and "rigid" $\mathcal{Q}_{ij}^{(r)}$ parts could be extracted. The "differential" part contains the correlations corresponding to the "old Boussinesq" relation (4.7). That is correlations arising from any shearing occuring in the fluid.

In the case of a global rotation with angular velocity Ω at the point $\mathbf{r}$ the mean flow is $\mathbf{u} = \mathbf{\Omega} \times \mathbf{r}$ and the resulting cross-correlations for the "differential" part are then

$$\mathcal{Q}_{r\phi}^{(d)} = -\nu_{\mathrm{T}} r \sin\theta \partial\Omega/\partial r, \qquad \mathcal{Q}_{\theta\phi}^{(d)} = -\nu_{\mathrm{T}} \sin\theta \partial\Omega/\partial\theta. \tag{4.17}$$

Rüdiger (1989) noted that it may seem the nonzero turbulent angular momentum transfer provided by the Boussinesq relations $\mathcal{Q}_{ij}^{(d)}$ can generate non-uniform rotation with ease. But the viscosity is effectively enhanced and, therefore, any non-uniform rotation is smoothed out more rapidly than if no turbulence were present. The existence of non-uniform rotation requires angular momentum transporters that do not vanish when the rotation is uniform. The meridional flow is one of these. In contrast to viscosity, it transports angular momentum even for a rigid rotation and for this reason necessarily sets up a differential rotation. The Reynolds stress in its Boussinesq formulation cannot create the necessary non-diffusive part of the turbulent angular momentum transfer. It has therefore come to be believed for a considerable time that Reynolds stresses generally have only a restricted significance for the theory of the non-uniform rotation.

Rüdiger (1989) studied what kind of convective pattern could imply a non-vanishing turbulent angular momentum transport in a stellar atmosphere. He derived geometrical constraints for non-vanishing cross-correlations taking into account the directions of gravity and rotation axis. He concluded, that completely isotropic fields can never posses cross-correlations. For anisotropic fields he got the result, that for a finite turbulent momentum transfer to occur, there have to be at least two preferred directions. In a rotating star the radial direction (gravity) and the direction of rotational axis are good candidates. Then the convective motions will be sufficiently complex.

Rüdiger (1989) noted that Biermann had argued in 1951 that the Boussinesq relations do not give the whole truth for anisotropic turbulence. For a quantitative estimate of the angular momentum flux Biermann had constructed a spectral function for the turbulence with two parts. The first part represented a completely isotropic motion, and the second motion was solely in the direction of gravity, $\mathbf{g}$. By considering the conservation of angular momentum flux in the equatorial plane he obtained a term proportional to Ω that creates the non-uniformity of the angular velocity Ω. This term survives even when uniform rotation is assumed. These considerarions lead to the fact that the rigid part can be written $\mathcal{Q}_{ij}^{(r)} = \Lambda_{ijk}\Omega_k$. The analogy with α-effect is strong. In certain kinds of turbulence, $\overline{\mathbf{u}' \times \mathbf{B}'}_i = \alpha_{ij}B_k$ correlations survive even when the prevailing magnetic fields are uniform (Rüdiger 1989).

Rüdiger (1989) studied the structure of the Λ-effect in more detail. To make tensor Λ_{ijk} anisotropic he chose as preferred direction the radial direction $\hat{\mathbf{g}}$. Besides $\hat{\mathbf{g}}$ only δ_{ij} and ϵ_{ijk} could be the used to construct the tensor Λ_{ijk}. By using some general symmetry and transformational properties of $\mathcal{Q}_{ij}$ and Λ_{ijk} Rüdiger (1989) obtained simple but, nevertheless, quite general expressions for the cross-correlations

$$\mathcal{Q}_{r\phi} = \sin\theta\left(-\nu_{vv} r\partial\Omega/\partial r + \Lambda_V\Omega\right), \qquad \mathcal{Q}_{\theta\phi} = -\sin\theta\nu_{hh}\partial\Omega/\partial\theta, \tag{4.18}$$

which are characterized by three free parameters. The anisotropic turbulence is reflected here also with the possibility that the turbulent viscosity may be different in vertical (ν_{vv}) and horizontal (ν_{hh}) directions. To take account also the latitude dependence the viscosities may be thought of as functions of $\sin^2\theta$ and may be written as

$$\left\{ \begin{array}{c} \nu_{vv} \\ \nu_{hh} \end{array} \right\} = \nu_{\mathrm{T}} \sum_{l=0} \left\{ \begin{array}{c} \nu_{vv}^{l} \\ \nu_{hh}^{l} \end{array} \right\} \sin^{2l}\theta. \tag{4.19}$$

More generally the Λ_{ijk}, used to derive the equations (4.18), may be regarded as the first term in an expansion for small Ω. Adding successively higher power of Ω to Λ_{ijk} the effect of higher rotation rates can be taken account. By including two $\boldsymbol{\Omega}$ vectors Rüdiger (1989) obtained

$$\mathcal{Q}_{r\phi}^{(r)} = \Lambda_V\Omega\sin\theta = \nu_{\mathrm{T}}V\Omega\sin\theta, \qquad \mathcal{Q}_{\theta\phi}^{(r)} = \Lambda_H\Omega\cos\theta = \nu_{\mathrm{T}}H\Omega\cos\theta. \tag{4.20}$$

Here the influence of anisotropic turbulence to viscosity is negleted and only one turbulent viscosity coefficient ν_{T} is used.

For a rotation that is not too slow, the Λ itself has a θ-dependence of the form

$$V = \sum_{l=0} V^{(l)}\sin^{2l}\theta, \qquad H = \sum_{l=1} H^{(l)}\sin^{2l}\theta \tag{4.21}$$

(Rüdiger 1989). The fundamental constant $V^{(0)}$ does not lead to latitudinally varying rotation, but it is the main factor determining the radial variation of the rotation, through the relation

$$\frac{d\,(\ln\Omega)}{d\,(\ln r)} \simeq V^{(0)}. \tag{4.22}$$

Positive $V^{(0)}$ leads to super-rotation, and negative $V^{(0)}$ to sub-rotation (Rüdiger 1989). Rüdiger derived in 1982 the rotation law $\Omega \sim$ constant$+\frac{1}{2}\sum_{l=1}\left(\mathrm{d}\ V^{(l)} + l^{-l}H^{(l)}\right)\sin^{2l}\theta$, which is for a surface layer of the convection zone of fractional thickness d. This shows that any Λ-mode with nonvanishing l generates latitudinally-dependent differential rotation. The observations only fit theories with $l = 1$, or at most $l = 2$, because terms of higher order than $\sin^4\theta$ do not occur with significant amplitude in the observed solar rotation law. Usually the Λ-effect is considered at most up to $l = 1$

$$\Lambda_V = \nu_\mathrm{T}\left(V^{(0)} + V^{(1)}\sin^2\theta\right), \qquad \Lambda_H = \nu_\mathrm{T}H^{(1)}\sin^2\theta. \tag{4.23}$$

Terms $V^{(0)}$, $V^{(1)}$, and $H^{(1)}$ are expected to be of the order unity. The terms $V^{(1)}$ and $H^{(1)}$ become important for larger angular velocities (Rüdiger 1989).

Rüdiger (1989) decomposed the turbulent correlation tensor in the case of spherical polar coordinates for the presentational purposes as

$$\begin{aligned}\mathcal{Q}_{ij} = {} & \begin{pmatrix} \overline{u_r'^2} & 0 & 0 \\ 0 & \overline{u_\theta'^2} & 0 \\ 0 & 0 & \overline{u_\phi'^2} \end{pmatrix} - \begin{pmatrix} 0 & 0 & \nu_\mathrm{T} r\partial\Omega/\partial r \\ 0 & 0 & \nu_\mathrm{T}\partial\Omega/\partial\theta \\ \nu_\mathrm{T} r\partial\Omega/\partial r & \nu_\mathrm{T}\partial\Omega/\partial\theta & 0 \end{pmatrix}\sin\theta \\ & + \begin{pmatrix} 0 & 0 & \Lambda_V\sin\theta \\ 0 & 0 & \Lambda_H\cos\theta \\ \Lambda_V\sin\theta & \Lambda_H\cos\theta & 0 \end{pmatrix}\Omega. \end{aligned} \tag{4.24}$$

This may be written in the more symbolic form $\mathcal{Q}_{ij} = \mathcal{Q}_{ij}^{(0)} + \mathcal{Q}_{ij}^{(d)} + \mathcal{Q}_{ij}^{(r)}$. The "original" turbulence $\mathcal{Q}_{ij}^{(0)}$ additionally included into the equation above is isotropic. It does not contain any contributions from anisotropic effects such as gravity, rotation or from any shearing flows found in stellar bodies (Rüdiger 1989). In the incompressible case the derivatives of diagonal elements of correlation tensor disappear so the contribution of $\mathcal{Q}_{ij}^{(0)}$ is negleted in this treatment.

4.5 Need for feedback mechanisms

The dynamo equation (4.11) is linear with respect to the magnetic field $\mathbf{B}$, so the solutions either grow or decay exponentially. For a critical value of the magnetic Reynolds number solutions which neither grow or decay are found. For growing solutions the magnetic field will be eventually strong enough to influence the flow via the Lorentz force. Depending on the scale of the motions affected by the Lorentz force term micro- or macro-feedback is used (Brandenburg 1990).

Finding suitable mean magnetic field $\mathbf{B}$ as a solution to the dynamo equation (4.11) with the help of a given mean velocity field $\mathbf{u}$ is called the kinematic dynamo or $\alpha\omega$-dynamo problem. The common study with kinematic dynamos is the influence of strengths and radial profiles of α- and ω-effects to the nature of the solutions. The action of differential rotation is called ω-effect. The usual procedure is to map the types of solutions on a C_α-C_Ω-plane. The dimensionless parameters $C_\alpha = \alpha_{max} R/\eta_{\mathrm{T}}$ and $C_\Omega = (\Delta\Omega)_{max} R^2/\eta_{\mathrm{T}}$ describe the strengths of α- and ω-effects, with $(\Delta\Omega)_{max}$ describing the maximum difference of stellar angular velocity Ω in the radial direction (Brandenburg 1990).

Usually considered micro-feedback is the influence of the magnetic field on the turbulent motions and on the helicity. In this way α is also affected, and this interaction in parameterized form can be written as $\alpha = \alpha_0 q(\mathbf{B})$, where q is a monotonously decreasing function between 1 and 0. This kind of nonlinear effect is called α-quenching mechanism. The α–effect becomes suppressed, when the large–scale magnetic field energy reaches equipartition with the energy of turbulent motions. Brandenburg (1990) lists some early investigations about this kind of quenching.

The dynamo equation (4.11) can also be solved together with the Reynolds equation (4.10) giving two macro-feedback effects in this system of equations. Firstly the magnetic back–reaction on the mean flow occurs via the Lorentz–force term $\mathbf{j} \times \mathbf{B}$ in the momentum equation and secondly the mean flow in turn influences the magnetic field via the Lorentz term $\mathbf{u} \times \mathbf{B}$ in the dynamo equation. The solving of the dynamo and Reynolds equations together is more selfconsistent procedure since the determination of differential rotation profiles for kinematic dynamos is quite arbitrary. This kind of dynamo including feedbacks between the dynamo and Reynolds equation for the large scale fields is called Malkus-Proctor dynamo after Malkus and Proctor (1975) who were the first ones to investigate the problem theoretically. Proctor (1977) started the numerical study of these kinds of dynamos. If the Λ-effect arising in the anisotropic stellar turbulence (Rüdiger 1989) is included into the Reynolds stresses the dynamo is called $\alpha\Lambda$-dynamo.

Jepps (1975) investigated the case of α-quenching where the magnetic quenching was a function of the strength of toroidal magnetic field. The assumption was that the magnetic field is mostly toroidal. Commonly the α-quenching is used in the form

$$q(\mathbf{B}) = \frac{1}{1 + f_{\mathrm{B}}\mathbf{B}^2}, \tag{4.25}$$

where $\mathbf{B}$ is the total magnetic field strength and f_{B} measures the relative contribution of the microscopic α–quenching and the macroscopic feedback introduced by the solution of the Navier–Stokes equation (e.g. Moss et al. 1991).

Also other possibilities in the category of the micro-feedback mechanisms has been proposed such as the Λ-quenching (Kitchatinov et al. 1994), viscosity quenching (Küker et al. 1996) and η-quenching (Rüdiger and Arlt 1996).

Observations show that the solar magnetic field is at least at the surface concentrated in flux tubes. Parker argued in 1975 that magnetic flux tubes in the solar convection zone may give rise to a rapid loss of flux on a time scale shorter than the solar magnetic cycle period. This has led many workers to elaborate specific dynamo models for the base of the solar convection zone where the magnetic fields could survive a time comparable to the length of the solar cycle. It has been argued that the buoancy could be the most important nonlinear mechanism in stars. However many arguments have also been proposed to overcome the buoyancy problem. The flux-tube dynamo approach introduced by Schüssler in 1980 incorborating the fibril nature of the magnetic fields leads finally to a dynamo equation similar to equation (4.14) (more detailed discussion contained in Brandenburg 1990).

Chapter 5

Earlier nonaxisymmetric and related research

Steenbeck, Krause and Rädler made break through in 1966 with the systematic formulation of the principles of mean-field electrodynamics including the discovery of the α-effect (Brandenburg 1990). Steenbeck and Krause (1969) developed axisymmetric $\alpha\omega$-dynamo models where the full radial extension is included with proper boundary conditions. Their results gave evidence for the possibility of magnetohydrodynamic alternating field dynamos. Some models could even produce migration of magnetic fields from the poles to the equator which is similar to the observed periodic migration pattern of sunspot belts. This was vividly demonstrated by constructing butterfly diagrams from the calculated magnetic fields. The migration pattern is the most basic feature of a solar magnetism that can be explained by axisymmetric mean–field dynamo models. From this time onwards a large amount of effort has been spent in improving the models of Steenbeck and Krause (1969) and in understanding the basic features in more detail.

Brandenburg et al. (1989) reviewed the earlier investigations of the possibility of non-axisymmetric modes of the magnetic field for $\alpha\omega$-dynamos and α^2-dynamos. In some $\alpha\omega$-models a slight preference of non-axisymmetric modes has been found for moderate magnitudes of differential rotation. In α^2-models a clear preference for non-axisymmetric modes occurs if particular anisotropies of the α-effect or related effects are taken into account. Brandenburg et al. (1989) could confirm these results for linear dynamos.

Rädler et al. (1990) made numerical investigations of the stability and time evolution of nonlinear (α-quenched) mean field dynamo models in three dimensions. They could confirm the results obtained with linear models previously. Model with an isotropic α-effect produced only axisymmetric modes of magnetic field. For a model with anisotropic α-effect also stable nonaxisymmetric modes were found. An $\alpha\omega$-dynamo model suggested that also "mixed" solutions, with axi- and nonaxisymmetric components both present simultaneously are possible.

Moss et al. (1991) studied in three spatial dimensions, the same time dependent, quasi-kinematic, nonlinear mean field dynamo equation as Rädler et al. (1990) did. The new numerical algorithm used was quite different though. The behaviour of the systems that they studied did depend quite sensitively both on the assumed profiles of differential rotation and α-effect, and also on their parameterized magnitudes. They could show that for a simple α-quenching nonlinearity together with suitable choices of underlying radial profiles of differential rotation and the α-effect, stable nonaxisymmetric solutions can be found by numerical integration. Also stable mixed solutions with axi- and nonaxisymmetric components both simultaneously present were found.

Barker (1993) solved the Reynolds equation considering only the fundamental Λ-mode (Rüdiger 1989) and negleting the magnetic field. The mean fluid velocity field obtained by this way was subsituted to the dynamo equation. Essentially this was a linear dynamo problem since the velocity field in the dynamo equation could be considered to be given. The turbulent electromotive force $\mathcal{E} = \alpha_0 \cos\theta \mathbf{B}$ was considered for simplicity with isotropic latitude dependent α-effect without introducing any micro-feedbacks. The critical dynamo number for the onset of dynamo action was determined for different hydrodynamic models for both axisymmetric and nonaxisymmetric magnetic fields.

Barker and Moss (1993) used essentially the same procedure as Barker (1993) i.e. the solution to the Reynolds equation was inserted to the dynamo equation. The most important difference was that the traditional form of α-quenching expressed by the equation (4.25) was in use. Again the result was that stable nonaxisymmetric magnetic fields could be generated.

Barker and Moss (1994) in turn solved the Reynolds equation and the dynamo equation simultaneously. The Λ-effect was negleted this time. The traditional form of α-quenching expressed by the equation (4.25) was in use. They found strikingly that the axisymmetric solutions are not stable in contrast to some previous calculations (Rädler et al. 1990; Moss et al. 1991). The stable nonaxisymmetric fields were symmetric with respect to the stellar equator.

Moss et al. (1995) also solved the Reynolds equation and the dynamo equation simultaneously. The Λ-effect (Rüdiger 1989) was considered in the simplest form with only the fundamental Λ-mode being nonzero. The turbulent electromotive force was considered with the traditional isotropic latitude dependent α-effect without any micro-feedback effects. The principal result was that, stable nonaxisymmetric magnetic fields can be exited at moderate values of the Taylor number for a range of values of the fundamental Λ-mode. These fields were symmetric with respect to the stellar equator.

Moss and Tuominen (1997) developed a simple mean field dynamo model and applied it to two dynamo-active corotating spheres that may be separate, touch or partially overlap to model components of a close or contact binary system. They assumed the dynamo action to occupy the entire volumes of these spheres, with the α-coefficient being a function of azimuthal coordinate measured about an axis parallel to the rotation axis. The geometry of the system is intrinsically nonaxisymmetric and tidal interactions can be expected to reduce severely the differential rotation and, to lock the spin and orbital frequencies. Tidal interaction was assumed to reduce differential rotation to such a degree that the dynamo action is of α^2 type. In this type of dynamo the ω-effect is neglible and the α-effect is solely responsible of the generation of any magnetic fields. The traditional form of α-effect with the usual latitudinal dependence and α-quenching nonlinearity expressed by the equation (4.25) was used. They also negleted the effects of any large-scale circulation. A variety of nonaxisymmetric magnetic fields were produced, steady and unsteady, of both odd, even and mixed parity (equation (7.2) defines the parity) with respect to the equatorial plane. Moss and Tuominen (1997) stressed that they modeled the simplest form of nonlinear dynamo without inclusion of any of the effects that have previously been shown to favour the growth of nonaxisymmetric fields. The geometry of their model is naturally nonaxisymmetric. Yet the solutions obtained reflected more than the obvious asymmetry of the system. Some solutions indicated a field maximum at the longitudes corresponding to the intersection of the line of centers with the stellar surfaces. This configuration is similar to the preferred longitudes of active regions on the surfaces of some RS CVn stars.

Chapter 6

Dynamo model

6.1 Equations

In the present work the standard mean field dynamo (6.1) and Reynolds equation (6.2)

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} + \alpha \mathbf{B} - \eta_{\mathrm{T}} \nabla \times \mathbf{B}), \tag{6.1}$$

$$\varrho \left[\frac{\partial}{\partial t} \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right] = -\nabla P + \mathbf{j} \times \mathbf{B} - \nabla \cdot \left(\varrho \mathcal{Q}^{(r)} \right) - \varrho \nu_{\mathrm{T}} \nabla \times \nabla \times \mathbf{u} \tag{6.2}$$

are solved in an spherical shell, $R_0 \leq r \leq R$, using polar coordinate system r, θ and ϕ corresponding to the radial, latitudinal and longitudinal directions, respectively. The conditions $\nabla \cdot \mathbf{B} = 0$, $\nabla \cdot \mathbf{u} = 0$, representing conservation of magnetic flux and mass in an incompressible fluid are supposed to be met. In the dynamo equation (6.1), $\mathbf{B}$ is the mean magnetic field, α is a scalar quantity describing the α–effect, and η_{T} is the turbulent diffusivity. In the Reynolds equation (6.2), ϱ is fluid density, $\mathbf{j} = \mu_0^{-1} \nabla \times \mathbf{B}$ is the current density, and $P = p + \varrho \psi_g$ is the reduced pressure including gravitational potential (Barker and Moss 1994; Moss et al. 1995). The decomposition of turbulent Reynolds stresses $\mathcal{Q} = \mathcal{Q}^{(r)} + \mathcal{Q}^{(d)}$, to the "rigid" $\mathcal{Q}^{(r)}$ and "differential" $\mathcal{Q}^{(d)}$ parts and the property $\nabla \cdot \mathcal{Q}^{(d)} = \nu_{\mathrm{T}} \nabla \times \nabla \times \mathbf{u} = \nu_{\mathrm{T}} \nabla^{\mathbf{2}} \mathbf{u}$ for an incompressible fluid are utilized in the Reynolds equation above (Rüdiger 1989).

6.2 Parameterization of turbulence

The simplest traditional form of the α–effect with the usual latitude dependence and the most usually used micro-feed back mechanism (quenching of small scale motions by the mean magnetic field) is used

$$\alpha = \frac{\alpha_0 \cos\theta}{1 + f_{\mathrm{B}} \mathbf{B}^2}, \tag{6.3}$$

where α_0 is the strength of the α–effect and θ is the latitude coordinate. The value of the constant f_{B} measures the relative contributions of the microscopic quenchings and the macroscopic feedbacks introduced by the solution of the Reynolds equation (Barker and Moss 1994).

Differential rotation is modeled by the simplest possible form of the Λ–effect

$$\mathcal{Q}_{ij}^{(r)} = \sin\theta \cdot \Omega \begin{pmatrix} 0 & 0 & \Lambda_V \\ 0 & 0 & 0 \\ \Lambda_V & 0 & 0 \end{pmatrix}, \tag{6.4}$$

where $\Lambda_V = \nu_{\mathrm{T}} V^{(0)}$. The fundamental lambda constant $V^{(0)}$ is expected to be of the order unity (Rüdiger 1989).

6.3 Numerical procedure

The axi– and nonaxisymmetric parts of the magnetic and velocity fields are separated by writing

$$\mathbf{B} = \nabla \times (a\hat{\phi}) + b\hat{\phi} + \nabla \times \nabla \times (\Phi\hat{r}) + \nabla \times (\Psi\hat{r}), \tag{6.5}$$

$$\mathbf{u} = \nabla \times (\psi\hat{\phi}) + v\hat{\phi} + \nabla \times \nabla \times (S\hat{r}) + \nabla \times (T\hat{r}), \tag{6.6}$$

where a, b, v and ψ are axisymmetric (independent of the longitudinal coordinate ϕ), and the nonaxisymmetric superpotential

$$\Phi\left(r,\theta,\phi\right) = \sum_{m=1}^{k} \Phi_m\left(r,\theta\right) e^{im\phi} \tag{6.7}$$

with similar expansions for ψ, S, T, and the reduced pressure P. After nondimensionalization the units are $x = r/R$, $\tau = t\eta_{\mathrm{T}}/R^2$, $\mathbf{B}^* = \mathbf{B}R/\left(\eta_{\mathrm{T}}^2\rho\mu_0\right)^{1/2}$, $\mathbf{u}^* = \mathbf{u}R/\eta_{\mathrm{T}}$ and $\alpha^* = \alpha R/\eta_{\mathrm{T}}$ for the spatial coordinate, time, magnetic field, velocity field and the α–effect, respectively. The nondimensional α–parameter, which is an input to the calculations, is then $C_\alpha = \alpha_0 R/\eta_{\mathrm{T}}$. The other important input parameter is the Taylor number $Ta = \left(2\Omega_0 R^2/\nu_{\mathrm{T}}\right)^2$ specifying the angular momentum. Here Ω_0 is the angular velocity of uniform rotation with the given angular momentum. The magnetic Prandtl number $P_{\mathrm{m}} = \nu_{\mathrm{T}}/\eta_{\mathrm{T}}$ is set to unity.

Integrated in time are the $k+1$ modal components of equations (6.1) and (6.2) using a modified Dufort–Frankel scheme, selecting $k = 3$. The numerical procedure is covered with more detail in Jepps (1975), Proctor (1977), Brandenburg et al. (1990), Moss et al. (1991), Barker and Moss (1994) and Moss et al. (1995). The spatial resolution 21×41 was used for small (C_α, Ta) and 41×81 for higher values.

Chapter 7

Results

7.1 Symmetry properties of large scale fields

Some measures are needed to quantify the overall appearance of a large scale field in the spherical rotating shell. Besides measures also some notation for various types of fields is needed. Rädler et al. (1990) discussed useful terminology and definitions for this study.

Any field can be decomposed into two parts that are symmetric or antisymmetric about the equatorial plane, and either part into its Fourier components with respect to the azimuthal coordinate. In that sense for example a magnetic field B is a superposition of its contributions B^m_{A} and B^m_{S} which have the form

$$B^m_{\mathrm{A,S}} = \Re\left(C^m_{\mathrm{A,S}} e^{im\phi}\right), \tag{7.1}$$

where C^m_{A} and C^m_{S} are complex vector fields that are antisymmetric or symmetric about the equatorial plane and symmetric about the axis of rotation, m is a non-negative integer, and ϕ the azimuthal coordinate. It is useful to adopt the usual notation and speak of Am or Sm parts, or Am or Sm fields.

Parity P measures the degree of symmetry or antisymmetry of a field about the equatorial plane by

$$P = \frac{E_{\mathrm{S}} - E_{\mathrm{A}}}{E_{\mathrm{S}} + E_{\mathrm{A}}}. \tag{7.2}$$

E_{S} and E_{A} are the energies associated with the parts of field symmetric and antisymmetric with respect to the rotational equator. If one substitutes $E_{\mathrm{A}} = 0$ in equation (7.2) one gets $P = 1$ for a symmetric (a Sm part dominates) field. Similarly $E_{\mathrm{S}} = 0$ gives $P = -1$ for a antisymmetric (a Am part dominates) field. Since $E = E_{\mathrm{S}} + E_{\mathrm{A}}$, then always $-1 \leq P \leq 1$.

The azimuthal symmetry parameter M measures the degree of deviation of a field from symmetry about the axis of rotation by

$$M = 1 - \frac{E_{\mathrm{ax}}}{E}. \tag{7.3}$$

E_{ax} is the energy in the axisymmetric part of the field and $E = E_{\mathrm{ax}} + E_{\mathrm{nx}}$ is the total energy of the field. Here the total energy is written as a sum of energies belonging to axisymmetric and nonaxisymmetric parts of field respectively. Considering that always $E_{\mathrm{ax}} \leq E$ from this follows that then also always $0 \leq M \leq 1$. Most importantly, if a field is axisymmetric then $M = 0$ and if a field is nonaxisymmetric then $M = 1$.

7.2 Discussion

Calculations were performed with $V^{(0)} = 1$ varying Taylor number from 10 to 5×10^5 and C_α from the critical value 10 (for Taylor number 100) to the far–supercritical value 1000. The depth of the convection zone was selected to be 0.5 R in all calculations corresponding K2 subgiant II Peg (Berdyugina et al. 1998b). In contrast to the earlier calculations (Moss et al. 1995) no smoothing was applied in α– or Λ–profiles in the bottom of the convection zone. The stability map is in Figure 7.1 showing the types of solutions obtained.

Figure 7.1: Stability map. With increasing C_α and Taylor number axisymmetric solutions change to nonaxisymmetric A1 solutions.

When C_α and Ta are small, the stable solutions are axisymmetric (S0 and A0). A0 solution corresponds to the solar magnetic field configuration. With increasing C_α and Ta, oscillatory nonaxisymmetric A1 solution becomes excited and is also the stable one. The $m = 1$ mode is dominating, but there is a weaker $m = 3$ mode present. This is in contrast with earlier calculations of Moss et al. (1995), who found the S1 solution to be stable in this parameter regime (they included only large–scale Lorentz force as a feedback). The magnetic field configuration is presented at the lower part of Figure 7.2. There are two magnetic "spots" of opposite polarity in both hemispheres. The oscillations seen in the various modes of magnetic energy in Figure 7.8 are caused by the simultaneous change of the shape and strength of the magnetic spots. No flip–flop type oscillations were found in the parameter range.

The Figures 7.3–7.8 give the parity P and symmetry M (equations (7.2) and (7.3)), for S0, A0 and A1 solutions. Also shown are the P–M diagrams and curves of axisymmetric magnetic energy E_M^0, nonaxisymmetric magnetic energy $\mathrm{E}_\mathrm{M}^1 + \mathrm{E}_\mathrm{M}^2 + \mathrm{E}_\mathrm{M}^3$ and various individual magnetic E_M^m and kinetic energy E_K^m modes (m) plotted as fuction of the nondimensional time τ.

Calculations with the used simple model show that nonaxisymmetric A1 solutions can dominate with moderate values of Taylor number. From the collected stellar imaging observations of II Peg (Berdyugina et al. 1998a,b) one image for December 1997 is presented at the upper part of Figure 7.2. A geometry of the excited field corresponds well with the spot images derived from the observations, concerning the existence of two active longitudes and the latitude correspondence. Further comparison of the observations and results of our theoretical model may be found in the reference Tuominen et al. (1999). Note that the southern hemisphere of the star is hidden, and hence the other two expected spots near the southern pole are invisible. The observed image does not give the magnetic polarity of the spots, polarimetric observations are needed to get that On the other hand, the model does not show the observed cyclic behaviour. A more detailed model with higher Taylor number could give such solutions.

7.3 Conclusions

In this treatment was shown that stable nonaxisymmetric magnetic fields are quite naturally exited in deep convective shells under fast stellar rotation, thought to be characteristic to the magnetically very active RS CVn stars. The solutions obtained resemble the strong and persistent nonaxisymmetric spot distribution observed on the RS CVn stars. Certainly the situation in these stars is rather more complex than anything produced by the used model: the "flip-flop" phenomenon, seen in many stars is currently not observed in the solutions produced by computer.

Figure 7.2: The A1 solution at the bottom resembles the observed surface images of II Peg at the top.

Figure 7.3: The P–M diagram and other related curves of the S0 solution plotted versus nondimensional time τ.

Figure 7.4: The magnetic (indice M) and kinetic (indice K) energy modes of the S0 solution plotted versus nondimensional time τ.

Figure 7.5: The P–M diagram and other related curves of the A0 solution plotted versus nondimensional time τ.

Figure 7.6: The magnetic (indice M) and kinetic (indice K) energy modes of the A0 solution plotted versus nondimensional time τ.

Figure 7.7: The P–M diagram and other related curves of the A1 solution plotted versus nondimensional time τ.

Figure 7.8: The magnetic (indice M) and kinetic (indice K) energy modes of the A1 solution plotted versus nondimensional time τ.

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